

जब तक आपको यह परीक्षण पुस्तिका खोलने को न कहा जाए तब तक न खोलें

टी.बी.सी. : RRAN-B-MTH

परीक्षण पुस्तिका अनुक्रम

क्रम संख्या

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परीक्षण पुस्तिका
गणित

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समय : दो घण्टे और तीस मिनट

पूर्णांक : 300

अनुदेश

1. परीक्षा प्रारम्भ होने के तुरन्त बाद, आप इस परीक्षण पुस्तिका की पड़ताल अवश्य कर लें कि इसमें कोई बिना छपा, फटा या छूटा हुआ पृष्ठ अथवा प्रश्नांश आदि न हो। यदि ऐसा है, तो इसे सही परीक्षण पुस्तिका से बदल लें।
2. कृपया ध्यान रखें कि OMR उत्तर-पत्रक में, उचित स्थान पर, रोल नम्बर और परीक्षण पुस्तिका अनुक्रम (सीरीज कोड) A, B, C या D को, ध्यान से एवं बिना किसी चूक या विसंगति के भरने और कूटबद्ध करने की ज़िम्मेदारी उम्मीदवार की है। किसी भी प्रकार की चूक/विसंगति की स्थिति में उत्तर-पत्रक निरस्त कर दिया जाएगा।
3. इस परीक्षण पुस्तिका पर साथ में दिए गए कोष्ठक में आपको अपना अनुक्रमांक लिखना है। परीक्षण पुस्तिका पर और कुछ न लिखें।
4. इस परीक्षण पुस्तिका में 120 प्रश्नांश (प्रश्न) दिए गए हैं। प्रत्येक प्रश्नांश हिन्दी और अंग्रेज़ी दोनों में छपा है। प्रत्येक प्रश्नांश में चार प्रत्युत्तर (उत्तर) दिए गए हैं। इनमें से एक प्रत्युत्तर को चुन लें, जिसे आप उत्तर-पत्रक पर अंकित करना चाहते हैं। यदि आपको ऐसा लगे कि एक से अधिक प्रत्युत्तर सही हैं, तो उस प्रत्युत्तर को अंकित करें जो आपको सर्वोत्तम लगे। प्रत्येक प्रश्नांश के लिए केवल एक ही प्रत्युत्तर चुनना है।
5. आपको अपने सभी प्रत्युत्तर अलग से दिए गए उत्तर-पत्रक पर ही अंकित करने हैं। उत्तर-पत्रक में दिए गए निर्देश देखें।
6. सभी प्रश्नांशों के अंक समान हैं।
7. इससे पहले कि आप परीक्षण पुस्तिका के विभिन्न प्रश्नांशों के प्रत्युत्तर उत्तर-पत्रक पर अंकित करना शुरू करें, आपको प्रवेश प्रमाण-पत्र के साथ प्रेषित अनुदेशों के अनुसार कुछ विवरण उत्तर-पत्रक में देने हैं।
8. आप अपने सभी प्रत्युत्तरों को उत्तर-पत्रक में भरने के बाद तथा परीक्षा के समापन पर केवल उत्तर-पत्रक अधीक्षक को सौंप दें। आपको अपने साथ परीक्षण पुस्तिका ले जाने की अनुमति है।
9. कच्चे काम के लिए पत्रक परीक्षण पुस्तिका के अन्त में संलग्न हैं।
10. गलत उत्तरों के लिए दण्ड :
वस्तुनिष्ठ प्रश्न-पत्रों में उम्मीदवार द्वारा दिए गए गलत उत्तरों के लिए दण्ड दिया जाएगा।
 - (i) प्रत्येक प्रश्न के लिए चार वैकल्पिक उत्तर हैं। उम्मीदवार द्वारा प्रत्येक प्रश्न के लिए दिए गए एक गलत उत्तर के लिए प्रश्न हेतु नियत किए गए अंकों का एक-तिहाई दण्ड के रूप में काटा जाएगा।
 - (ii) यदि कोई उम्मीदवार एक से अधिक उत्तर देता है, तो इसे गलत उत्तर माना जाएगा, यद्यपि दिए गए उत्तरों में से एक उत्तर सही होता है, फिर भी उस प्रश्न के लिए उपर्युक्तानुसार ही उसी तरह का दण्ड दिया जाएगा।
 - (iii) यदि उम्मीदवार द्वारा कोई प्रश्न हल नहीं किया जाता है, अर्थात् उम्मीदवार द्वारा उत्तर नहीं दिया जाता है, तो उस प्रश्न के लिए कोई दण्ड नहीं दिया जाएगा।

जब तक आपको यह परीक्षण पुस्तिका खोलने को न कहा जाए तब तक न खोलें

Note : English version of the instructions is printed on the back cover of this Booklet.

1. Let A and B be matrices of order 3×3 .

If $|A| = \frac{1}{2\sqrt{2}}$ and $|B| = \frac{1}{729}$, then what

is the value of $|2B(\text{adj}(3A))|$?

(a) 27

(b) $\frac{-27}{2\sqrt{2}}$

(c) $\frac{27}{2}$

(d) 1

2. If z is any complex number and

$iz^3 + z^2 - z + i = 0$, where $i = \sqrt{-1}$, then

what is the value of $(|z| + 1)^2$?

(a) 1

(b) 4

(c) 81

(d) 121

3. What is the sum of all four digit numbers formed by using all digits 0, 1, 4, 5 without repetition of digits?

(a) 44440

(b) 46460

(c) 46440

(d) 64440

4. If x , y and z are the cube roots of unity, then what is the value of $xy + yz + zx$?

(a) 0

(b) 1

(c) 2

(d) 3

5. A man has 7 relatives (4 women and 3 men). His wife also has 7 relatives (3 women and 4 men). In how many ways can they invite 3 women and 3 men so that 3 of them are man's relatives and 3 of them are his wife's relatives?

(a) 340

(b) 484

(c) 485

(d) 469

6. A triangle PQR is such that 3 points lie on the side PQ , 4 points on QR and 5 points on RP respectively. Triangles are constructed using these points as vertices. What is the number of triangles so formed?

(a) 205

(b) 206

(c) 215

(d) 220

7. If $\log_b a = p$, $\log_d c = 2p$ and $\log_f e = 3p$, then what is $(ace)^{\frac{1}{p}}$ equal to ?

- (a) bd^2f^3
- (b) bdf
- (c) b^3d^2f
- (d) $b^2d^2f^2$

8. If $-\sqrt{2}$ and $\sqrt{3}$ are roots of the equation $a_0 + a_1x + a_2x^2 + a_3x^3 + x^4 = 0$ where a_0, a_1, a_2, a_3 are integers, then which one of the following is correct ?

- (a) $a_2 = a_3 = 0$
- (b) $a_2 = 0$ and $a_3 = -5$
- (c) $a_0 = 6, a_3 = 0$
- (d) $a_1 = 0$ and $a_2 = 5$

9. Let z_1 and z_2 be two complex numbers

such that $\left| \frac{z_1 + z_2}{z_1 - z_2} \right| = 1$, then what is

$\operatorname{Re}\left(\frac{z_1}{z_2}\right) + 1$ equal to ?

- (a) -1
- (b) 0
- (c) 1
- (d) 5

10. If $26! = n8^k$, where k and n are positive integers, then what is the maximum value of k ?

- (a) 6
- (b) 7
- (c) 8
- (d) 9

11. Consider the following statements in respect of two non-singular matrices A and B of the same order n :

1. $\operatorname{adj}(AB) = (\operatorname{adj}A)(\operatorname{adj}B)$
2. $\operatorname{adj}(AB) = \operatorname{adj}(BA)$
3. $(AB)\operatorname{adj}(AB) - |AB|I_n$ is a null matrix of order n

How many of the above statements are correct ?

- (a) None
- (b) Only one statement
- (c) Only two statements
- (d) All three statements

12. Consider the following statements in respect of a non-singular matrix A of order n :

1. $A(\text{adj}A^T) = A(\text{adj}A)^T$
2. If $A^2 = A$, then A is identity matrix of order n
3. If $A^3 = A$, then A is identity matrix of order n

Which of the statements given above are correct ?

- (a) 1 and 2 only
- (b) 2 and 3 only
- (c) 1 and 3 only
- (d) 1, 2 and 3

13. How many four-digit natural numbers are there such that all of the digits are even ?

- (a) 625
- (b) 500
- (c) 400
- (d) 256

14. If $\omega \neq 1$ is a cube root of unity, then what are the solutions of $(z - 100)^3 + 1000 = 0$?

- (a) $10(1 - \omega)$, $10(10 - \omega^2)$, 100
- (b) $10(10 - \omega)$, $10(10 - \omega^2)$, 90

(c) $10(1 - \omega)$, $10(10 - \omega^2)$, 1000

(d) $(1 + \omega)$, $(10 + \omega^2)$, -1

15. What is $(1 + i)^4 + (1 - i)^4$ equal to, where $i = \sqrt{-1}$?

- (a) 4
- (b) 0
- (c) -4
- (d) -8

16. Consider the following statements in respect of a skew-symmetric matrix A of order 3 :

1. All diagonal elements are zero.
2. The sum of all the diagonal elements of the matrix is zero.
3. A is orthogonal matrix.

Which of the statements given above are correct ?

- (a) 1 and 2 only
- (b) 2 and 3 only
- (c) 1 and 3 only
- (d) 1, 2 and 3

17. Four digit numbers are formed by using the digits 1, 2, 3, 5 without repetition of digits. How many of them are divisible by 4?

- (a) 120
- (b) 24
- (c) 12
- (d) 6

18. What is the remainder when 2^{120} is divided by 7?

- (a) 1
- (b) 3
- (c) 5
- (d) 6

19. For what value of n is the determinant

$$\begin{vmatrix} C(9,4) & C(9,3) & C(10,n-2) \\ C(11,6) & C(11,5) & C(12,n) \\ C(m,7) & C(m,6) & C(m+1,n+1) \end{vmatrix} = 0$$

for every $m > n$?

- (a) 4
- (b) 5
- (c) 6
- (d) 7

20. If ABC is a triangle, then what is the value of the determinant

$$\begin{vmatrix} \cos C & \sin B & 0 \\ \tan A & 0 & \sin B \\ 0 & \tan(B+C) & \cos C \end{vmatrix} ?$$

- (a) -1
- (b) 0
- (c) 1
- (d) 3

21. What is the number of different matrices, each having 4 entries that can be formed using 1, 2, 3, 4 (repetition is allowed)?

- (a) 72
- (b) 216
- (c) 254
- (d) 768

22. Let $A = \{x \in \mathbb{R} : -1 < x < 1\}$. Which of the following is/are bijective functions from A to itself?

1. $f(x) = x|x|$
2. $g(x) = \cos(\pi x)$

Select the correct answer using the code given below:

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

23. Let R be a relation on the open interval $(-1, 1)$ and is given by

$R = \{(x, y) : |x + y| < 2\}$. Then which one of the following is correct ?

- (a) R is reflexive but neither symmetric nor transitive
- (b) R is reflexive and symmetric but not transitive
- (c) R is reflexive and transitive but not symmetric
- (d) R is an equivalence relation

24. For any three non-empty sets A, B, C , what is $(A \cup B) - \{(A - B) \cup (B - A) \cup (A \cap B)\}$ equal to ?

- (a) Null set
- (b) A
- (c) B
- (d) $(A \cup B) - (A \cap B)$

25. If a, b, c are the sides of triangle ABC , then what is

$$\begin{vmatrix} a^2 & b \sin A & c \sin A \\ b \sin A & 1 & \cos A \\ c \sin A & \cos A & 1 \end{vmatrix} \text{ equal to ?}$$

- (a) Zero
- (b) Area of triangle
- (c) Perimeter of triangle
- (d) $a^2 + b^2 + c^2$

26. If a, b, c are in AP; b, c, d are in GP; c, d, e are in HP, then which of the following is/are correct ?

1. a, c and e are in GP

2. $\frac{1}{a}, \frac{1}{c}, \frac{1}{e}$ are in GP

Select the correct answer using the code given below :

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

27. What is the number of solutions of $\log_4(x - 1) = \log_2(x - 3)$?

- (a) Zero
- (b) One
- (c) Two
- (d) Three

28. For $x \geq y > 1$,

$$\text{let } \log_x\left(\frac{x}{y}\right) + \log_y\left(\frac{y}{x}\right) = k,$$

then the value of k can never be equal to

- (a) -1
- (b) $-\frac{1}{2}$
- (c) 0
- (d) 1

29. If $A = \begin{bmatrix} \sin 2\theta & 2\sin^2 \theta - 1 & 0 \\ \cos 2\theta & 2\sin \theta \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then

which of the following statements is/are correct?

1. $A^{-1} = \text{adj}A$
2. A is skew-symmetric matrix
3. $A^{-1} = A^T$

Select the correct answer using the code given below:

- (a) 1 only
- (b) 1 and 2
- (c) 1 and 3
- (d) 2 and 3

30. What is the coefficient of x^{10} in the expansion of $(1-x^2)^{20} \left(2-x^2-\frac{1}{x^2}\right)^{-5}$?

- (a) -1
- (b) 1
- (c) 10
- (d) Coefficient of x^{10} does not exist

31. If the 4th term in the expansion of

$\left(mx + \frac{1}{x}\right)^n$ is $\frac{5}{2}$, then what is the value of mn ?

- (a) -3
- (b) 3
- (c) 6
- (d) 12

32. If a , b and c ($a > 0$, $c > 0$) are in GP, then consider the following in respect of the equation $ax^2 + bx + c = 0$:

1. The equation has imaginary roots.
2. The ratio of the roots of the equation is $1 : \omega$ where ω is a cube root of unity.
3. The product of roots of the equation is $\left(\frac{b^2}{a^2}\right)$.

Which of the statements given above are correct?

- (a) 1 and 2 only
- (b) 2 and 3 only
- (c) 1 and 3 only
- (d) 1, 2 and 3

33. If $x^2 + mx + n$ is an integer for all integral values of x , then which of the following is/are correct?

1. m must be an integer
2. n must be an integer

Select the correct answer using the code given below:

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

34. In a binomial expansion of $(x + y)^{2n+1}(x - y)^{2n+1}$, the sum of middle terms is zero. What

is the value of $\left(\frac{x^2}{y^2}\right)$?

- (a) 1
- (b) 2
- (c) 4
- (d) 8

35. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{6, 7\}$. What is the number of onto functions from A to B ?

- (a) 10
- (b) 20
- (c) 30
- (d) 32

36. What is $\frac{\sqrt{3} \cos 10^\circ - \sin 10^\circ}{\sin 25^\circ \cos 25^\circ}$ equal to?

- (a) 1
- (b) $\sqrt{3}$
- (c) 2
- (d) 4

37. What is $(\sin 9^\circ - \cos 9^\circ)$ equal to?

- (a) $-\frac{\sqrt{5-\sqrt{5}}}{2}$
- (b) $-\frac{\sqrt{5-\sqrt{3}}}{2}$
- (c) $\frac{\sqrt{5-\sqrt{5}}}{2}$
- (d) $\frac{\sqrt{5-\sqrt{5}}}{4}$

38. If in a triangle ABC , $\sin^3 A + \sin^3 B + \sin^3 C = 3 \sin A \sin B \sin C$, then what is

the value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$;

where a, b, c are sides of the triangle?

- (a) $a + b + c$
- (b) $ab + bc + ca$
- (c) $(a + b)(b + c)(c + a)$
- (d) 0

39. If $\cos^{-1}x = \sin^{-1}x$, then which one of the following is correct ?

(a) $x = 1$

(b) $x = \frac{1}{2}$

(c) $x = \frac{1}{\sqrt{2}}$

(d) $x = \frac{1}{\sqrt{3}}$

40. What is the number of solutions of $(\sin\theta - \cos\theta)^2 = 2$ where $-\pi < \theta < \pi$?

(a) Only one

(b) Only two

(c) Four

(d) No solution

41. ABC is a triangle such that angle $C = 60^\circ$, then what is $\frac{\cos A + \cos B}{\cos\left(\frac{A+B}{2}\right)}$

equal to ?

(a) 2

(b) $\sqrt{2}$

(c) 1

(d) $\frac{1}{\sqrt{2}}$

42. What is $\sqrt{15 + \cot^2\left(\frac{\pi}{4} - 2\cot^{-1}3\right)}$ equal to ?

(a) 1

(b) 7

(c) 8

(d) 16

43. What is the value of $\sin 10^\circ \cdot \sin 50^\circ + \sin 50^\circ \cdot \sin 250^\circ + \sin 250^\circ \cdot \sin 10^\circ$ equal to ?

(a) $-\frac{1}{4}$

(b) $-\frac{3}{4}$

(c) $\frac{3\sin 10^\circ}{4}$

(d) $-\frac{3\cos 10^\circ}{4}$

44. What is $\tan^{-1}\left(\frac{a}{b}\right) - \tan^{-1}\left(\frac{a-b}{a+b}\right)$ equal to ?

(a) $-\frac{\pi}{4}$

(b) $\frac{\pi}{4}$

(c) $\tan^{-1}\left(\frac{a^2 - b^2}{a^2 + b^2}\right)$

(d) $\tan^{-1}\left(\frac{2ab}{a^2 + b^2}\right)$

45. Under which one of the following conditions does the equation $(\cos\beta - 1)x^2 + (\cos\beta)x + \sin\beta = 0$ in x have a real root for $\beta \in [0, \pi]$?

- (a) $1 - \cos\beta < 0$
- (b) $1 - \cos\beta \leq 0$
- (c) $1 - \cos\beta > 0$
- (d) $1 - \cos\beta \geq 0$

46. In a triangle ABC , $AB = 16$ cm, $BC = 63$ cm and $AC = 65$ cm. What is the value of $\cos 2A + \cos 2B + \cos 2C$?

- (a) -1
- (b) 0
- (c) 1
- (d) $\frac{76}{65}$

47. If $f(\theta) = \frac{1}{1 + \tan\theta}$ and $\alpha + \beta = \frac{5\pi}{4}$, then what is the value of $f(\alpha)f(\beta)$?

- (a) $-\frac{1}{2}$
- (b) $\frac{1}{2}$
- (c) 1
- (d) 2

48. If $\tan\alpha$ and $\tan\beta$ are the roots of the equation $x^2 - 6x + 8 = 0$, then what is the value of $\cos(2\alpha + 2\beta)$?

- (a) $\frac{13}{75}$
- (b) $\frac{13}{85}$
- (c) $\frac{17}{85}$
- (d) $\frac{19}{85}$

49. What is the value of $\tan 65^\circ + 2\tan 45^\circ - 2\tan 40^\circ - \tan 25^\circ$?

- (a) 0
- (b) 1
- (c) 2
- (d) 4

50. Consider the following statements :

1. In a triangle ABC , if $\cot A \cdot \cot B \cdot \cot C > 0$, then the triangle is an acute angled triangle.
2. In a triangle ABC , if $\tan A \cdot \tan B \cdot \tan C > 0$, then the triangle is an obtuse angled triangle.

Which of the statements given above is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

51. If (a, b) is the centre and c is the radius of the circle $x^2 + y^2 + 2x + 6y + 1 = 0$, then what is the value of $a^2 + b^2 + c^2$?

- (a) 19
- (b) 18
- (c) 17
- (d) 11

52. If $(1, -1, 2)$ and $(2, 1, -1)$ are the end points of a diameter of a sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz - 1 = 0$, then what is $u + v + w$ equal to?

- (a) -2
- (b) -1
- (c) 1
- (d) 2

53. The number of points represented by the equation $x = 5$ on the xy -plane is

- (a) Zero
- (b) One
- (c) Two
- (d) Infinitely many

54. If $\langle l, m, n \rangle$ are the direction cosines of a normal to the plane $2x - 3y + 6z + 4 = 0$, then what is the value of $49(7l^2 + m^2 - n^2)$?

- (a) 0
- (b) 1
- (c) 3
- (d) 71

55. A line through $(1, -1, 2)$ with direction ratios $\langle 3, 2, 2 \rangle$ meets the plane $x + 2y + 3z = 18$. What is the point of intersection of line and plane?

- (a) $(4, 4, 1)$
- (b) $(2, 4, 1)$
- (c) $(4, 1, 4)$
- (d) $(3, 4, 7)$

56. If p is the perpendicular distance from origin to the plane passing through $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$, then what is $3p^2$ equal to?

- (a) 4
- (b) 3
- (c) 2
- (d) 1

57. If the direction cosines $\langle l, m, n \rangle$ of a line are connected by relation $l + 2m + n = 0$, $2l - 2m + 3n = 0$, then what is the value of $l^2 + m^2 - n^2$?

(a) $\frac{1}{101}$

(b) $\frac{29}{101}$

(c) $\frac{41}{101}$

(d) $\frac{92}{101}$

58. If a variable line passes through the point of intersection of the lines $x + 2y - 1 = 0$ and $2x - y - 1 = 0$ and meets the coordinate axes in A and B , then what is the locus of the mid-point of AB ?

(a) $3x + y = 10xy$

(b) $x + 3y = 10xy$

(c) $3x + y = 10$

(d) $x + 3y = 10$

59. What is the equation to the straight line passing through the point $(-\sin\theta, \cos\theta)$ and perpendicular to the line $x\cos\theta + y\sin\theta = 9$?

(a) $x\sin\theta - y\cos\theta - 1 = 0$

(b) $x\sin\theta - y\cos\theta + 1 = 0$

(c) $x\sin\theta - y\cos\theta = 0$

(d) $x\cos\theta - y\sin\theta + 1 = 0$

60. Two points P and Q lie on line $y = 2x + 3$. These two points P and Q are at a distance 2 units from another point $R(1, 5)$. What are the coordinates of the points P and Q ?

(a) $\left(1 + \frac{2}{\sqrt{5}}, 5 + \frac{4}{\sqrt{5}}\right), \left(1 - \frac{2}{\sqrt{5}}, 5 - \frac{4}{\sqrt{5}}\right)$

(b) $\left(3 + \frac{2}{\sqrt{5}}, 5 + \frac{4}{\sqrt{5}}\right), \left(-1 - \frac{2}{\sqrt{5}}, 5 - \frac{4}{\sqrt{5}}\right)$

(c) $\left(1 - \frac{2}{\sqrt{5}}, 5 + \frac{4}{\sqrt{5}}\right), \left(1 + \frac{2}{\sqrt{5}}, 5 - \frac{4}{\sqrt{5}}\right)$

(d) $\left(3 - \frac{2}{\sqrt{5}}, 5 + \frac{4}{\sqrt{5}}\right), \left(-1 + \frac{2}{\sqrt{5}}, 5 - \frac{4}{\sqrt{5}}\right)$

61. If two sides of a square lie on the lines $2x + y - 3 = 0$ and $4x + 2y + 5 = 0$, then what is the area of the square in square units?

(a) 6.05

(b) 6.15

(c) 6.25

(d) 6.35

62. ABC is a triangle with $A(3, 5)$. The mid-points of sides AB, AC are at $(-1, 2), (6, 4)$ respectively. What are the coordinates of centroid of the triangle ABC ?

(a) $\left(\frac{8}{3}, \frac{11}{3}\right)$

(b) $\left(\frac{7}{3}, \frac{7}{3}\right)$

(c) $\left(2, \frac{8}{3}\right)$

(d) $\left(\frac{8}{3}, 2\right)$

63. ABC is an acute angled isosceles triangle. Two equal sides AB and AC lie on the lines $7x - y - 3 = 0$ and $x + y - 5 = 0$. If θ is one of the equal angles, then what is $\cot\theta$ equal to?

(a) $\frac{1}{3}$

(b) $\frac{1}{2}$

(c) $\frac{2}{3}$

(d) 2

64. In the parabola $y^2 = 8x$, the focal distance of a point P lying on it is 8 units. Which of the following statements is/are correct?

1. The coordinates of P can be $(6, 4\sqrt{3})$.
2. The perpendicular distance of P from the directrix of parabola is 8 units.

Select the correct answer using the code given below :

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

65. What is the eccentricity of the ellipse if the angle between the straight lines joining the foci to an extremity of the minor axis is 90° ?

(a) $\frac{1}{3}$

(b) $\frac{1}{2}$

(c) $\frac{1}{\sqrt{3}}$

(d) $\frac{1}{\sqrt{2}}$

66. Let $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$. If $\vec{a} \times (\vec{b} \times \vec{a}) = \alpha\hat{i} - \beta\hat{j} + \gamma\hat{k}$, then what is the value of $\alpha + \beta + \gamma$?

(a) 8

(b) 7

(c) 6

(d) 1

67. If a vector of magnitude 2 units makes an angle $\frac{\pi}{3}$ with $2\hat{i}$, $\frac{\pi}{4}$ with $3\hat{j}$ and an acute angle θ with $4\hat{k}$, then what are the components of the vector ?

- (a) $(1, \sqrt{2}, 1)$
- (b) $(1, -\sqrt{2}, 1)$
- (c) $(1, -\sqrt{2}, -1)$
- (d) $(1, \sqrt{2}, -1)$

68. Consider the following in respect of moment of a force :

1. The moment of force about a point is independent of point of application of force.
2. The moment of a force about a line is a vector quantity.

Which of the statements given above is/are correct ?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

69. For any vector \vec{r} , what is

$(\vec{r} \cdot \hat{i})(\vec{r} \times \hat{i}) + (\vec{r} \cdot \hat{j})(\vec{r} \times \hat{j}) + (\vec{r} \cdot \hat{k})(\vec{r} \times \hat{k})$ equal to ?

- (a) $\vec{0}$
- (b) \vec{r}
- (c) $2\vec{r}$
- (d) $3\vec{r}$

70. Let \vec{a} and \vec{b} are two vectors of magnitude 4 inclined at an angle $\frac{\pi}{3}$, then what is the angle between \vec{a} and $\vec{a} - \vec{b}$?

- (a) $\frac{\pi}{2}$
- (b) $\frac{\pi}{3}$
- (c) $\frac{\pi}{4}$
- (d) $\frac{\pi}{6}$

71. Let $y_1(x)$ and $y_2(x)$ be two solutions of the differential equation $\frac{dy}{dx} = x$. If $y_1(0) = 0$ and $y_2(0) = 4$, then what is the number of points of intersection of the curves $y_1(x)$ and $y_2(x)$?

- (a) No point
- (b) One point
- (c) Two points
- (d) More than two points

72. The differential equation, representing the curve $y = e^x(a \cos x + b \sin x)$ where a and b are arbitrary constants, is

- (a) $\frac{d^2y}{dx^2} + 2y = 0$
- (b) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$
- (c) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$
- (d) $\frac{d^2y}{dx^2} + y = 0$

73. If $f(x) = ax - b$ and $g(x) = cx + d$ are such that $f(g(x)) = g(f(x))$, then which one of the following holds ?

- (a) $f(d) = g(b)$
- (b) $f(b) + g(d) = 0$
- (c) $f(a) + g(c) = 2a$
- (d) $f(d) + g(b) = 2d$

74. What is $\int_{-1}^1 (3\sin x - \sin 3x) \cos^2 x dx$ equal to ?

- (a) $-\frac{1}{4}$
- (b) 0
- (c) $\frac{1}{2}$
- (d) $\frac{1}{4}$

75. What are the order and degree respectively of the differential equation

$$\left\{ 2 - \left(\frac{dy}{dx} \right)^2 \right\}^{0.6} = \frac{d^2y}{dx^2} ?$$

- (a) 2, 2
- (b) 2, 3
- (c) 5, 2
- (d) 2, 5

76. If $\frac{dy}{dx} = 2e^x y^3$, $y(0) = \frac{1}{2}$ then what is $4y^2(2 - e^x)$ equal to ?

- (a) 1
- (b) 2
- (c) 3
- (d) 4

77. Let $p = \int_a^b f(x) dx$ and $q = \int_a^b |f(x)| dx$.

If $f(x) = e^{-x}$, then which one of the following is correct ?

- (a) $p = 2q$
- (b) $p = -q$
- (c) $4p = q$
- (d) $p = q$

78. What is $\int_0^{\frac{\pi}{2}} \frac{a + \sin x}{2a + \sin x + \cos x} dx$ equal to ?

- (a) $\frac{\pi}{4}$
- (b) $\frac{\pi}{2}$
- (c) 1
- (d) 0

79. The non-negative values of b for which the function $\frac{16x^3}{3} - 4bx^2 + x$ has neither maximum nor minimum in the range $x > 0$ is

- (a) $0 < b < 1$
- (b) $1 < b < 2$
- (c) $b > 2$
- (d) $0 \leq b < 1$

80. Which one of the following is correct

in respect of $f(x) = \frac{1}{\sqrt{|x|-x}}$ and

$$g(x) = \frac{1}{\sqrt{x-|x|}} ?$$

- (a) $f(x)$ has some domain and $g(x)$ has no domain
- (b) $f(x)$ has no domain and $g(x)$ has some domain
- (c) $f(x)$ and $g(x)$ have the same domain
- (d) $f(x)$ and $g(x)$ do not have any domain

Consider the following for the next two (02) items that follow :

Given that $\int \frac{3\cos x + 4\sin x}{2\cos x + 5\sin x} dx =$
 $\frac{\alpha x}{29} + \frac{\beta}{29} \ln|2\cos x + 5\sin x| + c$

81. What is the value of α ?

- (a) 7
- (b) 13
- (c) 17
- (d) 26

82. What is the value of β ?

- (a) 7
- (b) 13
- (c) 17
- (d) 26

Consider the following for the next two (02) items that follow :

Let $f(x) = \frac{x}{\ln x}$; ($x > 1$)

83. Consider the following statements :

- 1. $f(x)$ is increasing in the interval (e, ∞)
- 2. $f(x)$ is decreasing in the interval $(1, e)$
- 3. $9\ln 7 > 7\ln 9$

Which of the statements given above are correct ?

- (a) 1 and 2 only
- (b) 2 and 3 only
- (c) 1 and 3 only
- (d) 1, 2 and 3

84. Consider the following statements :

- 1. $f''(e) = \frac{1}{e}$
- 2. $f(x)$ attains local minimum value at $x = e$
- 3. A local minimum value of $f(x)$ is e

Which of the statements given above are correct ?

- (a) 1 and 2 only
- (b) 2 and 3 only
- (c) 1 and 3 only
- (d) 1, 2 and 3

Consider the following for the next **two (02)** items that follow :

Let $f(x)$ and $g(x)$ be two functions such that

$$g(x) = x - \frac{1}{x} \text{ and } f \circ g(x) = x^3 - \frac{1}{x^3}.$$

85. What is $g[f(x) - 3x]$ equal to ?

(a) $x^3 - \frac{1}{x^3}$

(b) $x^3 + \frac{1}{x^3}$

(c) $x^2 - \frac{1}{x^2}$

(d) $x^2 + \frac{1}{x^2}$

86. What is $f''(x)$ equal to ?

(a) $-\frac{2}{x^3}$

(b) $2x + \frac{2}{x^3}$

(c) $6x + 3$

(d) $6x$

Consider the following for the next **two (02)** items that follow :

Let $f(x) = |x| + 1$ and $g(x) = [x] - 1$, where $[.]$ is the greatest integer function.

$$\text{Let } h(x) = \frac{f(x)}{g(x)}$$

87. Consider the following statements :

1. $f(x)$ is differentiable for all $x < 0$
2. $g(x)$ is continuous at $x = 0.0001$
3. The derivative of $g(x)$ at $x = 2.5$ is 1

Which of the statements given above are correct ?

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

88. What is $\lim_{x \rightarrow 0^-} h(x) + \lim_{x \rightarrow 0^+} h(x)$ equal to ?

(a) $-\frac{3}{2}$

(b) $-\frac{1}{2}$

(c) $\frac{1}{2}$

(d) $\frac{3}{2}$

Consider the following for the next **two (02)** items that follow :

$$\text{Let } \varphi(a) = \int_a^{a+100\pi} |\sin x| dx$$

89. What is $\varphi(a)$ equal to ?

(a) 0

(b) a

(c) $100a$

(d) 200

90. What is $\varphi'(a)$ equal to ?

(a) 0

(b) π

(c) 100

(d) 200

Consider the following for the next **two (02)** items that follow :

A differentiable function $f(x)$ has a local maximum at $x = 0$. Let $y = 2f(x) + ax - b$.

91. Which of the following is/are correct ?

1. $f'(0) = 0$

2. $f''(0) < 0$

Select the correct answer using the code given below :

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

92. The function y has a relative maxima at $x = 0$ for

(a) $a > 0, b = 0$

(b) for all b and $a = 0$

(c) for all $b > 0$ only

(d) for all a and $b = 0$

Consider the following for the next **two (02)** items that follow :

Let $f(x) = |x - 1|$, $g(x) = [x]$ and $h(x) = f(x)g(x)$ where $[.]$ is greatest integer function.

93. What is $\int_{-1}^0 h(x)dx$ equal to ?

(a) $-\frac{3}{2}$

(b) -1

(c) 0

(d) $\frac{1}{2}$

94. What is $\int_0^2 h(x)dx$ equal to ?

(a) $-\frac{3}{2}$

(b) -1

(c) 0

(d) $\frac{1}{2}$

Consider the following for the next **two (02)** items that follow :

$$\text{Let } \int \frac{dx}{\sqrt{x+1} - \sqrt{x-1}} = \alpha(x+1)^{\frac{3}{2}} + \beta(x-1)^{\frac{3}{2}} + c$$

95. What is the value of α ?

(a) $\frac{1}{3}$

(b) $\frac{2}{3}$

(c) 1

(d) $\frac{4}{3}$

96. What is the value of β ?

(a) $-\frac{2}{3}$

(b) $-\frac{1}{3}$

(c) $\frac{1}{3}$

(d) $\frac{2}{3}$

Consider the following for the next **two (02)** items that follow :

The circle $x^2 + y^2 - 2x = 0$ is partitioned by line $y = x$ in two segments. Let A_1, A_2 be the areas of major and minor segments respectively.

97. What is the value of A_1 ?

- (a) $\frac{\pi - 2}{4}$
- (b) $\frac{\pi + 2}{4}$
- (c) $\frac{3\pi - 2}{4}$
- (d) $\frac{3\pi + 2}{4}$

98. What is the value of $\frac{2(A_1 + A_2)}{A_1 - 3A_2}$?

- (a) π
- (b) 1
- (c) -1
- (d) $-\pi$

Consider the following for the next **two (02)** items that follow :

$$\text{Let } 3f(x) + f\left(\frac{1}{x}\right) = \frac{1}{x} + 1$$

99. What is $f(x)$ equal to ?

- (a) $\frac{1}{8x} - \frac{x}{8} + \frac{1}{4}$
- (b) $\frac{3}{8x} - \frac{x}{8} + \frac{3}{4}$

$$(c) \frac{3}{8x} + \frac{x}{8} + \frac{1}{4}$$

$$(d) \frac{3}{8x} - \frac{x}{8} + \frac{1}{4}$$

100. What is $8 \int_1^2 f(x) dx$ equal to ?

- (a) $\ln(8\sqrt{e})$
- (b) $\ln(4\sqrt{e})$
- (c) $\ln 2$
- (d) $\ln 2 - 1$

101. A bag contains 5 black and 4 white balls. A man selects two balls at random. What is the probability that both of these are of the same colour ?

- (a) $\frac{1}{6}$
- (b) $\frac{5}{108}$
- (c) $\frac{4}{9}$
- (d) $\frac{5}{18}$

102. If a random variable (x) follows binomial distribution with mean 5 and variance 4, and $5^{23}P(X=3) = \lambda 4^\lambda$, then what is the value of λ ?

- (a) 3
- (b) 5
- (c) 23
- (d) 25

103. From data $(-4, 1)$, $(-1, 2)$, $(2, 7)$ and $(3, 1)$, the regression line of y on x is obtained as $y = a + bx$, then what is the value of $2a + 15b$?

- (a) 6
- (b) 11
- (c) 17
- (d) 21

104. Let $x + 2y + 1 = 0$ and $2x + 3y + 4 = 0$ are two lines of regression computed from some bivariate data. If θ is the acute angle between them, then what is the value of $488 \tan 3\theta$?

- (a) 191
- (b) 161
- (c) 131
- (d) 121

105. If two random variables X and Y are connected by relation

$$\frac{2X - 3Y}{5X + 4Y} = 4 \text{ and } X \text{ follows Binomial distribution with parameters } n = 10 \text{ and } p = \frac{1}{2}, \text{ then what is the variance of } Y?$$

- (a) $\frac{810}{361}$
- (b) $\frac{9}{19}$
- (c) $\frac{21}{361}$
- (d) $\frac{121}{361}$

106. If a , b , c are in HP, then what is

$$\frac{1}{b-a} + \frac{1}{b-c} \text{ equal to ?}$$

1. $\frac{2}{b}$

2. $\frac{1}{a} + \frac{1}{c}$

3. $\frac{1}{2} \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right)$

Select the correct answer using the code given below :

- (a) 1 only
- (b) 2 only
- (c) 3 only
- (d) 1, 2 and 3

107. An edible oil is sold at the rates 150, 200, 250, 300 rupees per litre in four consecutive years. Assuming that an equal amount of money is spent on oil by a family in every year during these years, what is the average price of oil in rupees (approximately) per litre?

- (a) 210
- (b) 220
- (c) 230
- (d) 240

108. If the letters of the word "TIRUPATI" are written down at random, then what is the probability that both Ts are always consecutive?

(a) $\frac{1}{2}$

(b) $\frac{1}{4}$

(c) $\frac{1}{7}$

(d) $\frac{1}{14}$

109. Let $m = 77^n$. The index n is given a positive integral value at random. What is the probability that the value of m will have 1 in the units place?

- (a) $\frac{1}{2}$
- (b) $\frac{1}{3}$
- (c) $\frac{1}{4}$
- (d) $\frac{1}{n}$

110. Three different numbers are selected at random from the first 15 natural numbers. What is the probability that the product of two of the numbers is equal to third number?

- (a) $\frac{1}{91}$
- (b) $\frac{2}{455}$
- (c) $\frac{1}{65}$
- (d) $\frac{6}{455}$

Consider the following for the next two (02) items that follow:

Let A and B be two events such that $P(A \cup B) \geq 0.75$ and $0.125 \leq P(A \cap B) \leq 0.375$.

111. What is the minimum value of $P(A) + P(B)$?

- (a) 0.625
- (b) 0.750
- (c) 0.825
- (d) 0.875

112. What is the maximum value of $P(A) + P(B)$?

- (a) 0.75
- (b) 1.125
- (c) 1.375
- (d) 1.625

Consider the following for the next two (02) items that follow:

A , B and C are three events such that $P(A) = 0.6$, $P(B) = 0.4$, $P(C) = 0.5$, $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$ and $P(A \cap B \cap C) = 0.2$ and $P(A \cup B \cup C) \geq 0.85$.

113. What is the minimum value of $P(B \cap C)$?

- (a) 0.1
- (b) 0.2
- (c) 0.35
- (d) 0.45

114. What is the maximum value of $P(B \cap C)$?

- (a) 0.1
- (b) 0.2
- (c) 0.35
- (d) 0.45

Consider the following for the next **two (02)** items that follow :

An unbiased coin is tossed n times. The probability of getting at least one tail is p and the probability of at least two tails is q and $p - q = \frac{5}{32}$.

115. What is the value of n ?

- (a) 4
- (b) 5
- (c) 6
- (d) 7

116. What is the value of $p + q$?

- (a) $\frac{57}{32}$
- (b) $\frac{53}{32}$
- (c) $\frac{51}{32}$
- (d) 1

Consider the following for the next **two (02)** items that follow :

x_i	1	2	3	...	n
f_i	1	2^{-1}	2^{-2}	...	$2^{-(n-1)}$

117. What is $\sum_i^n x_i f_i$ equal to ?

- (a) $\frac{2^{n+1} - n + 2}{2^{n-1}}$
- (b) $\frac{2^{n+1} - n - 2}{2^{n-1}}$

(c) $\frac{2^{n+1} + n + 2}{2^{n-1}}$

(d) $\frac{2^{n+1} - n - 2}{2^n}$

118. What is the mean of the distribution ?

(a) $\frac{2^{n+1} - n + 2}{2^n - 1}$

(b) $\frac{2^{n+1} - n - 2}{2^{n-1}}$

(c) $\frac{2^{n+1} - n - 2}{2^n - 1}$

(d) $\frac{2^{n+1} - n + 2}{2^n}$

Consider the following for the next **two (02)** items that follow :

The marks obtained by 10 students in a Statistics test are 24, 47, 18, 32, 19, 15, 21, 35, 50 and 41.

119. What is the mean deviation of the largest five observations ?

- (a) 4.8
- (b) 5.5
- (c) 6
- (d) 7.5

120. What is the variance of the largest five observations ?

- (a) 14.6
- (b) 21.8
- (c) 25.2
- (d) 46.8

SPACE FOR ROUGH WORK

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