

जब तक आपको यह परीक्षण पुस्तिका खोलने को न कहा जाए तब तक न खोलें

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परीक्षण पुस्तिका अनुक्रम

क्रम संख्या

1520141

परीक्षण पुस्तिका
गणित



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पूर्णांक : 300

अ नु दे श

1. परीक्षा प्रारम्भ होने के तुरन्त बाद आप इस परीक्षण पुस्तिका की पड़ताल अवश्य कर लें कि इसमें कोई बिना छपा, फटा या छूटा हुआ पृष्ठ अथवा प्रश्नांश आदि न हो। यदि ऐसा है, तो इसे सही परीक्षण पुस्तिका से बदल लें।
2. कृपया ध्यान रखें कि OMR उत्तर-पत्रक में उचित स्थान पर रोल नम्बर और परीक्षण पुस्तिका अनुक्रम A, B, C या D को ध्यान से एवं बिना किसी चूक या विसंगति के भरने और कूटबद्ध करने की जिम्मेदारी उम्मीदवार की है। किसी भी प्रकार की चूक/विसंगति की स्थिति में उत्तर-पत्रक निरस्त कर दिया जाएगा।
3. इस परीक्षण पुस्तिका पर साथ में दिए गए कोष्ठक में आपको अपना अनुक्रमांक लिखना है। परीक्षण पुस्तिका पर और कुछ न लिखें।
4. इस परीक्षण पुस्तिका में 120 प्रश्नांश (प्रश्न) दिए गए हैं। प्रत्येक प्रश्नांश हिन्दी और अंग्रेजी दोनों में छपा है। प्रत्येक प्रश्नांश में चार प्रत्युत्तर (उत्तर) दिए गए हैं। इनमें से एक प्रत्युत्तर को चुन लें, जिसे आप उत्तर-पत्रक पर अंकित करना चाहते हैं। यदि आपको ऐसा लगे कि एक से अधिक प्रत्युत्तर सही हैं, तो उस प्रत्युत्तर को अंकित करें जो आपको सर्वोत्तम लगे। प्रत्येक प्रश्नांश के लिए केवल एक ही प्रत्युत्तर चुनना है।
5. आपको अपने सभी प्रत्युत्तर अलग से दिए गए उत्तर-पत्रक पर ही अंकित करने हैं। उत्तर-पत्रक में दिए गए निर्देश देखें।
6. सभी प्रश्नांशों के अंक समान हैं।
7. इससे पहले कि आप परीक्षण पुस्तिका के विभिन्न प्रश्नांशों के प्रत्युत्तर उत्तर-पत्रक पर अंकित करना शुरू करें, आपको प्रवेश प्रमाण-पत्र के साथ प्रेषित अनुदेशों के अनुसार कुछ विवरण उत्तर-पत्रक में देने हैं।
8. आप अपने सभी प्रत्युत्तरों को उत्तर-पत्रक में भरने के बाद तथा परीक्षा के समापन पर केवल उत्तर-पत्रक अधीक्षक को सौंप दें। आपको अपने साथ परीक्षण पुस्तिका ले जाने की अनुमति है।
9. कच्चे काम के लिए पत्रक, परीक्षण पुस्तिका के अन्त में संलग्न हैं।
10. गलत उत्तरों के लिए दण्ड :
वस्तुनिष्ठ प्रश्न-पत्रों में उम्मीदवार द्वारा दिए गए गलत उत्तरों के लिए दण्ड दिया जाएगा।
(i) प्रत्येक प्रश्न के लिए चार वैकल्पिक उत्तर हैं। उम्मीदवार द्वारा प्रत्येक प्रश्न के लिए दिए गए एक गलत उत्तर के लिए प्रश्न हेतु नियत किए गए अंकों का एक-तिहाई दण्ड के रूप में काटा जाएगा।
(ii) यदि कोई उम्मीदवार एक से अधिक उत्तर देता है, तो इसे गलत उत्तर माना जाएगा, यद्यपि दिए गए उत्तरों में से एक उत्तर सही होता है, फिर भी उस प्रश्न के लिए उपर्युक्तानुसार ही उसी तरह का दण्ड दिया जाएगा।
(iii) यदि उम्मीदवार द्वारा कोई प्रश्न हल नहीं किया जाता है अर्थात् उम्मीदवार द्वारा उत्तर नहीं दिया जाता है, तो उस प्रश्न के लिए कोई दण्ड नहीं दिया जाएगा।

जब तक आपको यह परीक्षण पुस्तिका खोलने को न कहा जाए तब तक न खोलें

Note : English version of the instructions is printed on the back cover of this Booklet.

1. If $x^2 + x + 1 = 0$, then what is the value of $x^{199} + x^{200} + x^{201}$?

(a) -1
(b) 0
(c) 1
(d) 3

2. If x, y, z are in GP, then which of the following is/are correct?

1. $\ln(3x), \ln(3y), \ln(3z)$ are in AP
2. $xyz + \ln(x), xyz + \ln(y), xyz + \ln(z)$ are in HP

Select the correct answer using the code given below.

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

3. If $\log_{10} 2, \log_{10}(2^x - 1), \log_{10}(2^x + 3)$ are in AP, then what is x equal to?

(a) 0
(b) 1
(c) $\log_2 5$
(d) $\log_5 2$

4. Let $S = \{2, 3, 4, 5, 6, 7, 9\}$. How many different 3-digit numbers (with all digits different) from S can be made which are less than 500?

(a) 30
(b) 49
(c) 90
(d) 147

5. If $p = (1111 \dots \text{up to } n \text{ digits})$, then what is the value of $9p^2 + p$?

(a) $10^n p$
(b) $2p \cdot 10^n$
(c) $10^n p - 1$
(d) $10^n p + 1$

6. The quadratic equation

$$3x^2 - (k^2 + 5k)x + 3k^2 - 5k = 0$$

has real roots of equal magnitude and opposite sign. Which one of the following is correct?

(a) $0 < k < \frac{5}{3}$
(b) $0 < k < \frac{3}{5}$ only
(c) $\frac{3}{5} < k < \frac{5}{3}$
(d) No such value of k exists

7. If $a_n = n(n!)$, then what is

$$a_1 + a_2 + a_3 + \dots + a_{10}$$

equal to?

- (a) $10! - 1$
- (b) $11! + 1$
- (c) $10! + 1$
- (d) $11! - 1$

8. If p and q are the non-zero roots of the equation $x^2 + px + q = 0$, then how many possible values can q have?

- (a) Nil
- (b) One
- (c) Two
- (d) Three

9. If

$$\Delta = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

then what is

$$\begin{vmatrix} 3d+5g & 4a+7g & 6g \\ 3e+5h & 4b+7h & 6h \\ 3f+5i & 4c+7i & 6i \end{vmatrix}$$

equal to?

- (a) Δ
- (b) 7Δ
- (c) 72Δ
- (d) -72Δ

10. If

$$\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$$

are in HP, then which of the following is/are correct?

- 1. a, b, c are in AP
- 2. $(b+c)^2, (c+a)^2, (a+b)^2$ are in GP

Select the correct answer using the code given below.

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

11. If

$$A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$$

where $a \in \mathbb{N}$, then what is

$$A^{100} - A^{50} - 2A^{25}$$

equal to?

- (a) $-2I$
- (b) $-I$
- (c) $2I$
- (d) I

where I is the identity matrix.

12. If

$$\begin{vmatrix} a & -b & a-b-c \\ -a & b & -a+b-c \\ -a & -b & -a-b+c \end{vmatrix} - kabc = 0$$

$(a \neq 0, b \neq 0, c \neq 0)$

then what is the value of k ?

- (a) -4
- (b) -2
- (c) 2
- (d) 4

13. What is $\sum_{n=1}^{8n+7} i^n$ equal to, where $i = \sqrt{-1}$?

- (a) -1
- (b) 1
- (c) i
- (d) $-i$

14. If $z = x + iy$, where $i = \sqrt{-1}$, then what does the equation $z\bar{z} + |z|^2 + 4(z + \bar{z}) - 48 = 0$ represent?

- (a) Straight line
- (b) Parabola
- (c) Circle
- (d) Pair of straight lines

15. Which one of the following is a square root of $2a + 2\sqrt{a^2 + b^2}$, where $a, b \in \mathbb{R}$?

- (a) $\sqrt{a+ib} + \sqrt{a-ib}$
- (b) $\sqrt{a+ib} - \sqrt{a-ib}$
- (c) $2a + ib$
- (d) $2a - ib$

where $i = \sqrt{-1}$.

16. If $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2 + bx + c = 0$, then which one of the following is correct?

- (a) $a^2 + b^2 - 2ac = 0$
- (b) $-a^2 + b^2 + 2ac = 0$
- (c) $a^2 - b^2 + 2ac = 0$
- (d) $a^2 + b^2 + 2ac = 0$

17. If $C(n, 4)$, $C(n, 5)$ and $C(n, 6)$ are in AP, then what is the value of n ?

- (a) 7
- (b) 8
- (c) 9
- (d) 10

18. How many 4-letter words (with or without meaning) containing two vowels can be constructed using only the letters (without repetition) of the word 'LUCKNOW'?

- (a) 240
- (b) 200
- (c) 150
- (d) 120

19. Suppose 20 distinct points are placed randomly on a circle. Which of the following statements is/are correct?

1. The number of straight lines that can be drawn by joining any two of these points is 380.
2. The number of triangles that can be drawn by joining any three of these points is 1140.

Select the correct answer using the code given below.

- (a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

20. How many terms are there in the expansion of

$$\left(\frac{a^2}{b^2} + \frac{b^2}{a^2} + 2\right)^{21}$$

where $a \neq 0$, $b \neq 0$?

- (a) 21
(b) 22
(c) 42
(d) 43

21. For what values of k is the system of equations

$$2k^2x + 3y - 1 = 0, \quad 7x - 2y + 3 = 0, \\ 6kx + y + 1 = 0$$

consistent?

- (a) $\frac{3 \pm \sqrt{11}}{10}$
(b) $\frac{21 \pm \sqrt{161}}{10}$
(c) $\frac{3 \pm \sqrt{7}}{10}$
(d) $\frac{4 \pm \sqrt{11}}{10}$

22. The inverse of a matrix A is given by

$$\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$$

What is A equal to?

- (a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
(b) $\begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$
(d) $\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$

23. What is the period of the function $f(x) = \ln(2 + \sin^2 x)$?

- (a) $\frac{\pi}{2}$
(b) π
(c) 2π
(d) 3π

24. If $\sin(A+B) = 1$ and $2\sin(A-B) = 1$, where $0 < A, B < \frac{\pi}{2}$, then what is $\tan A : \tan B$ equal to?

- (a) 1:2
- (b) 2:1
- (c) 1:3
- (d) 3:1

25. Consider a regular polygon with 10 sides. What is the number of triangles that can be formed by joining the vertices which have no common side with any of the sides of the polygon?

- (a) 25
- (b) 50
- (c) 75
- (d) 100

26. Consider all the real roots of the equation $x^4 - 10x^2 + 9 = 0$. What is the sum of the absolute values of the roots?

- (a) 4
- (b) 6
- (c) 8
- (d) 10

27. Consider the expansion of $(1+x)^n$. Let p, q, r and s be the coefficients of first, second, n th and $(n+1)$ th terms respectively. What is $(ps+qr)$ equal to?

- (a) $1+2n$
- (b) $1+2n^2$
- (c) $1+n^2$
- (d) $1+4n$

28. Let

$$\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$$

for $0 \leq x, y, z \leq 1$. What is the value of $x^{1000} + y^{1001} + z^{1002}$?

- (a) 0
- (b) 1
- (c) 3
- (d) 6

29. Let $\sin x + \sin y = \cos x + \cos y$ for all $x, y \in \mathbb{R}$. What is $\tan\left(\frac{x}{2} + \frac{y}{2}\right)$ equal to?

- (a) 1
- (b) 2
- (c) $\sqrt{2}$
- (d) $2\sqrt{2}$

30. Let

$$A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} \text{ and } (mI + nA)^2 = A$$

where m, n are positive real numbers and I is the identity matrix. What is $(m+n)$ equal to?

- (a) 0
- (b) $\frac{1}{2}$
- (c) 1
- (d) $\frac{3}{2}$

31. What is the value of the following?

$$\cot\left[\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right]$$

(a) $\frac{6}{17}$

(b) $\frac{7}{16}$

(c) $\frac{16}{7}$

(d) $\frac{17}{6}$

32. Let $4\sin^2 x = 3$, where $0 \leq x \leq \pi$. What is $\tan 3x$ equal to?

(a) -2

(b) -1

(c) 0

(d) 1

33. Let p , q and 3 be respectively the first, third and fifth terms of an AP. Let d be the common difference. If the product (pq) is minimum, then what is the value of d ?

(a) 1

(b) $\frac{3}{8}$

(c) $\frac{9}{8}$

(d) $\frac{9}{4}$

34. Consider the following statements in respect of the roots of the equation $x^3 - 8 = 0$:

1. The roots are non-collinear.

2. The roots lie on a circle of unit radius.

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

35. Let the equation $\sec x \cdot \operatorname{cosec} x = p$ have a solution, where p is a positive real number. What should be the smallest value of p ?

(a) $\frac{1}{2}$

(b) 1

(c) 2

(d) Minimum does not exist

36. For what value of θ , where $0 < \theta < \frac{\pi}{2}$, does $\sin \theta + \sin \theta \cos \theta$ attain maximum value?

(a) $\frac{\pi}{2}$

(b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$

(d) $\frac{\pi}{6}$

37. Consider the following statements in respect of sets :

1. The union over intersection of sets is distributive.
2. The complement of union of two sets is equal to intersection of their complements.
3. If the difference of two sets is equal to empty set, then the two sets must be equal.

Which of the above statements are correct?

- (a) 1 and 2 only
- (b) 2 and 3 only
- (c) 1 and 3 only
- (d) 1, 2 and 3

38. Consider three sets X , Y and Z having 6, 5 and 4 elements respectively. All these 15 elements are distinct. Let $S = (X - Y) \cup Z$. How many proper subsets does S have?

- (a) 255
- (b) 256
- (c) 1023
- (d) 1024

39. Consider the following statements in respect of relations and functions :

1. All relations are functions but all functions are not relations.
2. A relation from A to B is a subset of Cartesian product $A \times B$.
3. A relation in A is a subset of Cartesian product $A \times A$.

Which of the above statements are correct?

- (a) 1 and 2 only
- (b) 2 and 3 only
- (c) 1 and 3 only
- (d) 1, 2 and 3

40. If $\log_{10} 2 \log_2 10 + \log_{10}(10^x) = 2$, then what is the value of x ?

- (a) 0
- (b) 1
- (c) $\log_2 10$
- (d) $\log_5 2$

41. Let ABC be a triangle. If

$$\cos 2A + \cos 2B + \cos 2C = -1$$

then which one of the following is correct?

- (a) $\sin A \sin B \sin C = 0$
- (b) $\sin A \sin B \cos C = 0$
- (c) $\cos A \sin B \sin C = 0$
- (d) $\cos A \cos B \cos C = 0$

42. What is the value of the following determinant?

$$\begin{vmatrix} \cos C & \tan A & 0 \\ \sin B & 0 & -\tan A \\ 0 & \sin B & \cos C \end{vmatrix}$$

- (a) -1
(b) 0
(c) $2 \tan A \sin B \sin C$
(d) $-2 \tan A \sin B \sin C$

43. Suppose set A consists of first 250 natural numbers that are multiples of 3 and set B consists of first 200 even natural numbers. How many elements does $A \cup B$ have?

- (a) 324
(b) 364
(c) 384
(d) 400

44. Let S_k denote the sum of first k terms of an AP. What is $\frac{S_{30}}{S_{20} - S_{10}}$ equal to?

- (a) 1
(b) 2
(c) 3
(d) 4

45. If the roots of the equation

$$4x^2 - (5k + 1)x + 5k = 0$$

differ by unity, then which one of the following is a possible value of k ?

- (a) -3
(b) -1
(c) $-\frac{1}{5}$
(d) $-\frac{3}{5}$

46. Consider the digits 3, 5, 7, 9. What is the number of 5-digit numbers formed by these digits in which each of these four digits appears?

(a) 240

(b) 180

(c) 120

(d) 60

47. How many distinct matrices exist with all four entries taken from $\{1, 2\}$?

(a) 16

(b) 24

(c) 32

(d) 48

48. If $i = \sqrt{-1}$, then how many values does i^{-2n} have for different $n \in \mathbb{Z}$?

(a) One

(b) Two

(c) Four

(d) Infinite

49. If

$$x = \frac{a}{b-c}, \quad y = \frac{b}{c-a}, \quad z = \frac{c}{a-b}$$

then what is the value of the following?

$$\begin{vmatrix} 1 & -x & x \\ 1 & 1 & -y \\ 1 & z & 1 \end{vmatrix}$$

(a) 0

(b) 1

(c) abc

(d) $ab + bc + ca$

50. Consider the following in respect of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

1. Inverse of A does not exist

2. $A^3 = A$

3. $3A = A^2$

Which of the above are correct?

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

Direction : Consider the following for the next **two (02)** items that follow.

A circle is passing through the points $(5, -8)$, $(-2, 9)$ and $(2, 1)$.

51. What are the coordinates of the centre of the circle?

(a) $(-2, -50)$

(b) $(-50, -20)$

(c) $(-24, -58)$

(d) $(-58, -24)$

52. If r is the radius of the circle, then which one of the following is correct?

(a) $r < 10$

(b) $10 < r < 30$

(c) $30 < r < 60$

(d) $r > 60$

Direction : Consider the following for the next **two (02)** items that follow.

The two vertices of an equilateral triangle are $(0, 0)$ and $(2, 2)$.

53. Consider the following statements :

1. The third vertex has at least one irrational coordinate.
2. The area is irrational.

Which of the above statements is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

54. The difference of coordinates of the third vertex is

- (a) 0
- (b) $\sqrt{3}$
- (c) $2\sqrt{2}$
- (d) $2\sqrt{3}$

Direction : Consider the following for the next **two (02)** items that follow.

The coordinates of three consecutive vertices of a parallelogram $ABCD$ are $A(1, 3)$, $B(-1, 2)$ and $C(3, 5)$.

55. What is the equation of the diagonal BD ?

- (a) $2x - 3y + 2 = 0$
- (b) $3x - 2y + 5 = 0$
- (c) $2x - 3y + 8 = 0$
- (d) $3x - 2y - 5 = 0$

56. What is the area of the parallelogram?

- (a) 1 square unit
- (b) $\frac{3}{2}$ square units
- (c) 2 square units
- (d) $\frac{5}{2}$ square units

Direction : Consider the following for the next **two (02)** items that follow.

The equations of the sides AB , BC and CA of a triangle ABC are $x - 2 = 0$, $y + 1 = 0$ and $x + 2y - 4 = 0$ respectively.

57. What is the equation of the altitude through B on AC ?

- (a) $x - 3y + 1 = 0$
- (b) $x - 3y + 4 = 0$
- (c) $2x - y + 4 = 0$
- (d) $2x - y - 5 = 0$

58. What are the coordinates of circumcentre of the triangle?

- (a) $(4, 0)$
- (b) $(2, 1)$
- (c) $(0, 4)$
- (d) $(2, -1)$

Direction : Consider the following for the next **two (02)** items that follow.

The two ends of the latus rectum of a parabola are $(-2, 4)$ and $(-2, -4)$.

59. What is the maximum number of parabolas that can be drawn through these two points as end points of latus rectum?

- (a) Only one
- (b) Two
- (c) Four
- (d) Infinite

60. Consider the following statements in respect of such parabolas :

1. One of the parabolas passes through the origin $(0, 0)$.
2. The focus of one of the parabolas lies at $(-2, 0)$.

Which of the above statements is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

61. The locus of a point $P(x, y, z)$ which moves in such a way that $z = 7$ is a

- (a) line parallel to x -axis
- (b) line parallel to y -axis
- (c) line parallel to z -axis
- (d) plane parallel to xy -plane

62. Consider the following statements :

1. A line in space can have infinitely many direction ratios.
2. It is possible for certain line that the sum of the squares of direction cosines can be equal to sum of its direction cosines.

Which of the above statements is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

63. The xy -plane divides the line segment joining the points $(-1, 3, 4)$ and $(2, -5, 6)$

- (a) internally in the ratio $2:3$
- (b) internally in the ratio $3:2$
- (c) externally in the ratio $2:3$
- (d) externally in the ratio $2:1$

64. The number of spheres of radius r touching the coordinate axes is

- (a) 4
- (b) 6
- (c) 8
- (d) infinite

65. $ABCDEFGH$ is a cuboid with base $ABCD$.

Let $A(0, 0, 0)$, $B(12, 0, 0)$, $C(12, 6, 0)$ and $G(12, 6, 4)$ be the vertices. If α is the angle between AB and AG ; β is the angle between AC and AG , then what is the value of $\cos 2\alpha + \cos 2\beta$?

- (a) $\frac{40}{49}$
- (b) $\frac{64}{49}$
- (c) $\frac{120}{49}$
- (d) $\frac{160}{49}$

66. Let \vec{a} , \vec{b} and \vec{c} be unit vectors such that $\vec{a} \times \vec{b}$ is perpendicular to \vec{c} . If θ is the angle between \vec{a} and \vec{b} , then which of the following is/are correct?

1. $\vec{a} \times \vec{b} = \sin \theta \vec{c}$
2. $\vec{a} \cdot (\vec{b} \times \vec{c}) = 0$

Select the correct answer using the code given below.

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

67. If $\vec{a} + 3\vec{b} = 3\hat{i} - \hat{j}$ and $2\vec{a} + \vec{b} = \hat{i} - 2\hat{j}$, then what is the angle between \vec{a} and \vec{b} ?

- (a) 0
- (b) $\frac{\pi}{6}$
- (c) $\frac{\pi}{3}$
- (d) $\frac{\pi}{2}$

68. If $(\vec{a} + \vec{b})$ is perpendicular to \vec{a} and magnitude of \vec{b} is twice that of \vec{a} , then what is the value of $(4\vec{a} + \vec{b}) \cdot \vec{b}$ equal to?

- (a) 0
- (b) 1
- (c) $8|\vec{a}|^2$
- (d) $8|\vec{b}|^2$

69. Let \vec{a} , \vec{b} and \vec{c} be three vectors such that \vec{a} , \vec{b} and \vec{c} are coplanar. Which of the following is/are correct?

1. $(\vec{a} \times \vec{b}) \times \vec{c}$ is coplanar with \vec{a} and \vec{b}
2. $(\vec{a} \times \vec{b}) \times \vec{c}$ is perpendicular to $\vec{a} \times \vec{b}$

Select the correct answer using the code given below.

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

70. If the position vectors of A and B are $(\sqrt{2} - 1)\hat{i} - \hat{j}$ and $\hat{i} + (\sqrt{2} + 1)\hat{j}$ respectively, then what is the magnitude of \overrightarrow{AB} ?

(a) $2\sqrt{2}$

(b) $3\sqrt{2}$

(c) $2\sqrt{3}$

(d) $3\sqrt{3}$

71. If

$$y = (1+x)(1+x^2)(1+x^4)(1+x^8)(1+x^{16})$$

then what is $\frac{dy}{dx}$ at $x = 0$ equal to?

(a) 0

(b) 1

(c) 2

(d) 4

72. If $y = \cos x \cdot \cos 4x \cdot \cos 8x$, then what is

$$\frac{1}{y} \frac{dy}{dx} \text{ at } x = \frac{\pi}{4} \text{ equal to?}$$

(a) -1

(b) 0

(c) 1

(d) 3

73. Let $f(x)$ be a polynomial function such that $f \circ f(x) = x^4$. What is $f'(1)$ equal to?

(a) 0

(b) 1

(c) 2

(d) 4

74. What is

$$\lim_{n \rightarrow \infty} \frac{a^n + b^n}{a^n - b^n}$$

where $a > b > 1$, equal to?

(a) -1

(b) 0

(c) 1

(d) Limit does not exist

75. Let

$$f(x) = \begin{cases} 1 + \frac{x}{2k}, & 0 < x < 2 \\ kx, & 2 \leq x < 4 \end{cases}$$

If $\lim_{x \rightarrow 2} f(x)$ exists, then what is the value of k ?

(a) -2

(b) -1

(c) 0

(d) 1

76. Consider the following statements in respect of $f(x) = |x| - 1$:

1. $f(x)$ is continuous at $x = 1$.
2. $f(x)$ is differentiable at $x = 0$.

Which of the above statements is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

77. If $f(x) = \frac{[x]}{|x|}$, $x \neq 0$, where $[\cdot]$ denotes the greatest integer function, then what is the right-hand limit of $f(x)$ at $x = 1$?

- (a) -1
- (b) 0
- (c) 1
- (d) Right-hand limit of $f(x)$ at $x = 1$ does not exist

78. Consider the following statements in respect of the function

$$f(x) = \sin\left(\frac{1}{x^2}\right), \quad x \neq 0 :$$

1. It is continuous at $x = 0$, if $f(0) = 0$.
2. It is continuous at $x = \frac{2}{\sqrt{\pi}}$.

Which of the above statements is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

79. What is the range of the function $f(x) = 1 - \sin x$ defined on entire real line?

- (a) $(0, 2)$
- (b) $[0, 2]$
- (c) $(-1, 1)$
- (d) $[-1, 1]$

80. What is the slope of the tangent of $y = \cos^{-1}(\cos x)$ at $x = -\frac{\pi}{4}$?

- (a) -1
- (b) 0
- (c) 1
- (d) 2

81. What is the integral of $f(x) = 1 + x^2 + x^4$ with respect to x^2 ?

- (a) $x + \frac{x^3}{3} + \frac{x^5}{5} + C$
- (b) $\frac{x^3}{3} + \frac{x^5}{5} + C$
- (c) $x^2 + \frac{x^4}{4} + \frac{x^6}{6} + C$
- (d) $x^2 + \frac{x^4}{2} + \frac{x^6}{3} + C$

82. Consider the following statements in respect of the function $f(x) = x^2 + 1$ in the interval $(1, 2)$:

1. The maximum value of the function is 5.
2. The minimum value of the function is 2.

Which of the above statements is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

83. If $f(x)$ satisfies $f(1) = f(4)$, then what is

$\int_1^4 f'(x) dx$ equal to?

- (a) -1
- (b) 0
- (c) 1
- (d) 2

84. What is $\int_0^{\frac{\pi}{2}} e^{\ln(\cos x)} dx$ equal to?

- (a) -1
- (b) 0
- (c) 1
- (d) 2

85. If $\int \sqrt{1 - \sin 2x} dx = A \sin x + B \cos x + C$, where $0 < x < \frac{\pi}{4}$, then which one of the following is correct?

- (a) $A + B = 0$
- (b) $A + B - 2 = 0$
- (c) $A + B + 2 = 0$
- (d) $A + B - 1 = 0$

86. What is the order of the differential equation of all ellipses whose axes are along the coordinate axes?

- (a) 1
- (b) 2
- (c) 3
- (d) 4

87. What is the degree of the differential equation of all circles touching both the coordinate axes in the first quadrant?

- (a) 1
- (b) 2
- (c) 3
- (d) 4

88. What is the differential equation of

$$y = A - \frac{B}{x}?$$

(a) $xy_2 + y_1 = 0$

(b) $xy_2 + 2y_1 = 0$

(c) $xy_2 - 2y_1 = 0$

(d) $2xy_2 + y_1 = 0$

89. What is $\int_0^{\pi} \ln\left(\tan \frac{x}{2}\right) dx$ equal to?

(a) 0

(b) $\frac{1}{2}$

(c) 1

(d) 2

90. Where does the tangent to the curve $y = e^x$ at the point (0, 1) meet x-axis?

(a) (1, 0)

(b) (-1, 0)

(c) (2, 0)

(d) $\left(-\frac{1}{2}, 0\right)$

91. Consider the following statements in respect of the function $f(x) = x + \frac{1}{x}$:

1. The local maximum value of $f(x)$ is less than its local minimum value.

2. The local maximum value of $f(x)$ occurs at $x = 1$.

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

92. What is the maximum area of a rectangle that can be inscribed in a circle of radius 2 units?

(a) 4 square units

(b) 6 square units

(c) 8 square units

(d) 16 square units

93. What is $\int \frac{dx}{x(x^2 + 1)}$ equal to?

(a) $\frac{1}{2} \ln\left(\frac{x^2}{x^2 + 1}\right) + C$

(b) $\ln\left(\frac{x^2}{x^2 + 1}\right) + C$

(c) $\frac{3}{2} \ln\left(\frac{x^2}{x^2 + 1}\right) + C$

(d) $\frac{1}{2} \ln\left(\frac{x^2 + 1}{x^2}\right) + C$

94. What is the derivative of e^{e^x} with respect to e^x ?

- (a) e^{e^x}
- (b) e^x
- (c) $e^{e^x} e^x$
- (d) ee^x

95. What is the condition that $f(x) = x^3 + x^2 + kx$ has no local extremum?

- (a) $4k < 1$
- (b) $3k > 1$
- (c) $3k < 1$
- (d) $3k \leq 1$

96. If $f(x) = 2^x$, then what is $\int_2^{10} \frac{f'(x)}{f(x)} dx$ equal to?

- (a) $4 \ln 2$
- (b) $\ln 4$
- (c) $\ln 5$
- (d) $8 \ln 2$

97. If $\int_{-2}^0 f(x) dx = k$, then $\int_{-2}^0 |f(x)| dx$ is

- (a) less than k
- (b) greater than k
- (c) less than or equal to k
- (d) greater than or equal to k

98. If the function $f(x) = x^2 - kx$ is monotonically increasing in the interval $(1, \infty)$, then which one of the following is correct?

- (a) $k < 2$
- (b) $2 < k < 3$
- (c) $3 < k < 4$
- (d) $k > 4$

99. What is the area bounded by $y = [x]$, where $[\cdot]$ is the greatest integer function, the x -axis and the lines $x = -1.5$ and $x = -1.8$?

- (a) 0.3 square unit
- (b) 0.4 square unit
- (c) 0.6 square unit
- (d) 0.8 square unit

100. The tangent to the curve $x^2 = y$ at $(1, 1)$ makes an angle θ with the positive direction of x -axis. Which one of the following is correct?

- (a) $\theta < \frac{\pi}{6}$
- (b) $\frac{\pi}{6} < \theta < \frac{\pi}{4}$
- (c) $\frac{\pi}{4} < \theta < \frac{\pi}{3}$
- (d) $\frac{\pi}{3} < \theta < \frac{\pi}{2}$

101. Consider the following relations for two events E and F :

1. $P(E \cap F) \geq P(E) + P(F) - 1$
2. $P(E \cup F) = P(E) + P(F) + P(E \cap F)$
3. $P(E \cup F) \leq P(E) + P(F)$

Which of the above relations is/are correct?

- (a) 1 only
- (b) 3 only
- (c) 1 and 3 only
- (d) 1, 2 and 3

102. If $P(A|B) < P(A)$, then which one of the following is correct?

- (a) $P(B|A) < P(B)$
- (b) $P(B|A) > P(B)$
- (c) $P(B|A) = P(B)$
- (d) $P(B|A) > P(A)$

103. When the measure of central tendency is available in the form of mean, which one of the following is the most reliable and accurate measure of variability?

- (a) Range
- (b) Mean deviation
- (c) Standard deviation
- (d) Quartile deviation

104. A problem is given to three students A , B and C , whose probabilities of solving the problem independently are $\frac{1}{2}$, $\frac{3}{4}$ and p respectively. If the probability that the problem can be solved is $\frac{29}{32}$, then what is the value of p ?

- (a) $\frac{2}{5}$
- (b) $\frac{2}{3}$
- (c) $\frac{1}{3}$
- (d) $\frac{1}{4}$

105. In a cricket match, a batsman hits a six 8 times out of 60 balls he plays. What is the probability that on a ball played he does not hit a six?

- (a) $\frac{2}{3}$
- (b) $\frac{1}{15}$
- (c) $\frac{2}{15}$
- (d) $\frac{13}{15}$

Direction : Consider the following for the next **two (02)** items that follow.

Two regression lines are given as $3x - 4y + 8 = 0$ and $4x - 3y - 1 = 0$.

106. Consider the following statements :

1. The regression line of y on x is $y = \frac{3}{4}x + 2$.
2. The regression line of x on y is $x = \frac{3}{4}y + \frac{1}{4}$.

Which of the above statements is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

107. Consider the following statements :

1. The coefficient of correlations r is $\frac{3}{4}$.
2. The means of x and y are 3 and 4 respectively.

Which of the above statements is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

Direction : Consider the following for the next **two (02)** items that follow.

The marks obtained by 60 students in a certain subject out of 75 are given below :

Marks	Number of students
15-20	4
20-25	5
25-30	11
30-35	6
35-40	5
40-45	8
45-50	9
50-55	6
55-60	4
60-65	2

108. What is the median?

- (a) 35
- (b) 38
- (c) 39
- (d) 40

109. What is the mode?

- (a) 27.27
- (b) 27.73
- (c) 27.93
- (d) 28.27

110. What is the mean of natural numbers contained in the interval $[15, 64]$?

- (a) 36.8
- (b) 38.3
- (c) 39.5
- (d) 40.3

111. For the set of numbers

$$x, x, x+2, x+3, x+10$$

where x is a natural number, which of the following is/are correct?

1. Mean $>$ Mode
2. Median $>$ Mean

Select the correct answer using the code given below.

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

112. The mean of 10 observations is 5.5. If each observation is multiplied by 4 and subtracted from 44, then what is the new mean?

- (a) 20
- (b) 22
- (c) 34
- (d) 44

113. If g is the geometric mean of 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, then which one of the following is correct?

- (a) $8 < g < 16$
- (b) $16 < g < 32$
- (c) $32 < g < 64$
- (d) $g > 64$

114. If the harmonic mean of 60 and x is 48, then what is the value of x ?

- (a) 32
- (b) 36
- (c) 40
- (d) 44

115. What is the mean deviation of first 10 even natural numbers?

- (a) 5
- (b) 5.5
- (c) 10
- (d) 10.5

116. If

$$\sum_{i=1}^{10} x_i = 110 \quad \text{and} \quad \sum_{i=1}^{10} x_i^2 = 1540$$

then what is the variance?

- (a) 22
- (b) 33
- (c) 44
- (d) 55

117. 3-digit numbers are formed using the digits 1, 3, 7 without repetition of digits. A number is randomly selected. What is the probability that the number is divisible by 3?

- (a) 0
- (b) $\frac{1}{3}$
- (c) $\frac{1}{4}$
- (d) $\frac{1}{8}$

118. What is the probability that the roots of the equation $x^2 + x + n = 0$ are real, where $n \in \mathbb{N}$ and $n < 4$?

- (a) 0
- (b) $\frac{1}{4}$
- (c) $\frac{1}{3}$
- (d) $\frac{1}{2}$

119. If A and B are two events such that

$$P(\text{not } A) = \frac{7}{10}, \quad P(\text{not } B) = \frac{3}{10} \quad \text{and}$$

$$P(A|B) = \frac{3}{14}, \quad \text{then what is } P(B|A)$$

equal to?

- (a) $\frac{11}{14}$
- (b) $\frac{9}{14}$
- (c) $\frac{1}{4}$
- (d) $\frac{1}{2}$

120. Seven white balls and three black balls are randomly placed in a row. What is the probability that no two black balls are placed adjacently?

- (a) $\frac{7}{15}$
- (b) $\frac{8}{15}$
- (c) $\frac{11}{15}$
- (d) $\frac{13}{15}$

SPACE FOR ROUGH WORK

SPACE FOR ROUGH WORK



TEST BOOKLET
MATHEMATICS

1550141

INSTRUCTIONS

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THE TEST BOOKLET DOES NOT HAVE ANY UNPRINTED OR UNFOLDED PAGES.
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As stated here, it is the candidate's responsibility to ensure and fill in the test booklet
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the printed or handwritten in the test booklet. Any handwritten or printed
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