N.D.A & N.A EXAMINATION (II) 2018

जब तक आपको यह परीक्षण पुस्तिका खोलने को न कहा जाए तब तक न खोलें

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क्रम संख्या

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परीक्षण पुस्तिका अनुक्रम

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परीक्षण पुस्तिका

गणित

समय : दो घण्टे और तीस मिनट

पूर्णांक : 300

अनुदेश

1. परीक्षा प्रारम्भ होने के तुरन्त बाद, आप इस परीक्षण पुस्तिका की पड़ताल अवश्य कर लें कि इसमें कोई बिना छपा, फटा या छूटा हुआ पृष्ठ अथवा प्रश्नांश, आदि न हो । यदि ऐसा है, तो इसे सही परीक्षण पुस्तिका से बदल लीजिए ।
2. कृपया ध्यान रखें कि OMR उत्तर-पत्रक में, उचित स्थान पर, रोल नम्बर और परीक्षण पुस्तिका अनुक्रम (सीरीज कोड) A, B, C या D को, ध्यान से एवं बिना किसी चूक या विसंगति के भरने और कूटबद्ध करने की ज़िम्मेदारी उम्मीदवार की है । किसी भी प्रकार की चूक/विसंगति की स्थिति में उत्तर-पत्रक निरस्त कर दिया जाएगा ।
3. इस परीक्षण पुस्तिका पर साथ में दिए गए कोष्ठक में आपको अपना अनुक्रमांक लिखना है । परीक्षण पुस्तिका पर *और कुछ न* लिखें ।
4. इस परीक्षण पुस्तिका में **120** प्रश्नांश (प्रश्न) दिए गए हैं । प्रत्येक प्रश्नांश *हिन्दी* और *अंग्रेज़ी* दोनों में छपा है । प्रत्येक प्रश्नांश में चार प्रत्युत्तर (उत्तर) दिए गए हैं । इनमें से एक प्रत्युत्तर को चुन लें, जिसे आप उत्तर-पत्रक पर अंकित करना चाहते हैं । यदि आपको ऐसा लगे कि एक से अधिक प्रत्युत्तर सही हैं, तो उस प्रत्युत्तर को अंकित करें जो आपको सर्वोत्तम लगे । प्रत्येक प्रश्नांश के लिए *केवल एक ही* प्रत्युत्तर चुनना है ।
5. आपको अपने सभी प्रत्युत्तर अलग से दिए गए उत्तर-पत्रक पर *ही* अंकित करने हैं । उत्तर-पत्रक में दिए गए निर्देश देखिए ।
6. *सभी* प्रश्नांशों के अंक समान हैं ।
7. इससे पहले कि आप परीक्षण पुस्तिका के विभिन्न प्रश्नांशों के प्रत्युत्तर उत्तर-पत्रक पर अंकित करना शुरू करें, आपको प्रवेश प्रमाण-पत्र के साथ प्रेषित अनुदेशों के अनुसार कुछ विवरण उत्तर-पत्रक में देने हैं ।
8. आप अपने सभी प्रत्युत्तरों को उत्तर-पत्रक में भरने के बाद तथा परीक्षा के समापन पर *केवल उत्तर-पत्रक* अधीक्षक को सौंप दें । आपको अपने साथ परीक्षण पुस्तिका ले जाने की अनुमति है ।
9. कच्चे काम के लिए पत्रक परीक्षण पुस्तिका के अंत में संलग्न हैं ।
10. **ग़लत उत्तरों के लिए दण्ड :**

वस्तुनिष्ठ प्रश्न-पत्रों में उम्मीदवार द्वारा दिए गए ग़लत उत्तरों के लिए दण्ड दिया जाएगा ।

(i) प्रत्येक प्रश्न के लिए चार वैकल्पिक उत्तर हैं । उम्मीदवार द्वारा प्रत्येक प्रश्न के लिए दिए गए एक ग़लत उत्तर के लिए प्रश्न हेतु नियत किए गए अंकों का **एक-तिहाई** दण्ड के रूप में काटा जाएगा ।

(ii) यदि कोई उम्मीदवार एक से अधिक उत्तर देता है, तो इसे **ग़लत उत्तर** माना जाएगा, यद्यपि दिए गए उत्तरों में से एक उत्तर सही होता है, फिर भी उस प्रश्न के लिए उपर्युक्तानुसार ही उसी तरह का दण्ड दिया जाएगा ।

(iii) यदि उम्मीदवार द्वारा कोई प्रश्न हल नहीं किया जाता है, अर्थात् उम्मीदवार द्वारा उत्तर नहीं दिया जाता है, तो उस प्रश्न के लिए **कोई दण्ड नहीं** दिया जाएगा ।

जब तक आपको यह परीक्षण पुस्तिका खोलने को न कहा जाए तब तक न खोलें

Note : English version of the instructions is printed on the back cover of this Booklet.

1. What is the value of $\log_7 \log_7 \sqrt{7\sqrt{7\sqrt{7}}}$ equal to ?

(a) $3 \log_2 7$

(b) $1 - 3 \log_2 7$

(c) $1 - 3 \log_7 2$

(d) $\frac{7}{8}$

2. If an infinite GP has the first term x and the sum 5, then which one of the following is correct ?

(a) $x < -10$

(b) $-10 < x < 0$

(c) $0 < x < 10$

(d) $x > 10$

3. Consider the following expressions :

1. $x + x^2 - \frac{1}{x}$

2. $\sqrt{ax^2 + bx + x - c + \frac{d}{x} - \frac{e}{x^2}}$

3. $3x^2 - 5x + ab$

4. $\frac{2}{x^2 - ax + b^3}$

5. $\frac{1}{x} - \frac{2}{x+5}$

Which of the above are rational expressions ?

(a) 1, 4 and 5 only

(b) 1, 3, 4 and 5 only

(c) 2, 4 and 5 only

(d) 1 and 2 only

4. A square matrix A is called orthogonal if

(a) $A = A^2$

(b) $A' = A^{-1}$

(c) $A = A^{-1}$

(d) $A = A'$

where A' is the transpose of A.

5. If A, B and C are subsets of a Universal set, then which one of the following is ***not*** correct ?

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(b) $A' \cup (A \cup B) = (B' \cap A)' \cup A$

(c) $A' \cup (B \cup C) = (C' \cap B)' \cap A'$

(d) $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

where A' is the complement of A.

6. Let x be the number of integers lying between 2999 and 8001 which have at least two digits equal. Then x is equal to

(a) 2480

(b) 2481

(c) 2482

(d) 2483

7. The sum of the series $3 - 1 + \frac{1}{3} - \frac{1}{9} + \ldots$ is equal to

(a) $\frac{20}{9}$

(b) $\frac{9}{20}$

(c) $\frac{9}{4}$

(d) $\frac{4}{9}$

Consider the information given below and answer the two (02) items that follow :

A survey was conducted among 300 students. It was found that 125 students like to play cricket, 145 students like to play football and 90 students like to play tennis. 32 students like to play exactly two games out of the three games.

8. How many students like to play all the three games ?

(a) 14

(b) 21

(c) 28

(d) 35

9. How many students like to play exactly only one game ?

(a) 196

(b) 228

(c) 254

(d) 268

10. If α and β ($\neq 0$) are the roots of the quadratic equation $x^2 + \alpha x - \beta = 0$, then the quadratic expression $-x^2 + \alpha x + \beta$ where $x \in \mathbf{R}$ has

(a) Least value $-\frac{1}{4}$

(b) Least value $-\frac{9}{4}$

(c) Greatest value $\frac{1}{4}$

(d) Greatest value $\frac{9}{4}$

11. What is the coefficient of the middle term in the binomial expansion of $(2 + 3x)^4$?

(a) 6

(b) 12

(c) 108

(d) 216

12. For a square matrix A, which of the following properties hold ?

1. $(A^{-1})^{-1} = A$
2. $\det(A^{-1}) = \frac{1}{\det A}$
3. $(\lambda A)^{-1} = \lambda A^{-1}$ where λ is a scalar

Select the correct answer using the code given below :

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

13. Which one of the following factors does the expansion of the determinant $\begin{vmatrix} x & y & 3 \\ x^2 & 5y^3 & 9 \\ x^3 & 10y^5 & 27 \end{vmatrix}$ contain ?

(a) $x - 3$

(b) $x - y$

(c) $y - 3$

(d) $x - 3y$

14. What is the adjoint of the matrix

$$\begin{pmatrix} \cos(-\theta) & -\sin(-\theta) \\ -\sin(-\theta) & \cos(-\theta) \end{pmatrix}$$?

(a) $\begin{pmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

(b) $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

(c) $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

(d) $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

15. What is the value of

$$\left(\frac{-1+i\sqrt{3}}{2}\right)^{3n} + \left(\frac{-1-i\sqrt{3}}{2}\right)^{3n},$$

where $i = \sqrt{-1}$?

(a) 3

(b) 2

(c) 1

(d) 0

16. There are 17 cricket players, out of which 5 players can bowl. In how many ways can a team of 11 players be selected so as to include 3 bowlers ?

(a) C (17, 11)

(b) C (12, 8)

(c) C (17, 5) × C (5, 3)

(d) C (5, 3) × C (12, 8)

17. What is the value of $\log_9 27 + \log_8 32$?

(a) $\frac{7}{2}$

(b) $\frac{19}{6}$

(c) 4

(d) 7

18. If A and B are two invertible square matrices of same order, then what is $(AB)^{-1}$ equal to ?

(a) $B^{-1}A^{-1}$

(b) $A^{-1}B^{-1}$

(c) $B^{-1}A$

(d) $A^{-1}B$

19. If $a + b + c = 0$, then one of the solutions of

$$\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0 \text{ is}$$

(a) $x = a$

(b) $x = \sqrt{\frac{3(a^2+b^2+c^2)}{2}}$

(c) $x = \sqrt{\frac{2(a^2+b^2+c^2)}{3}}$

(d) $x = 0$

20. What should be the value of x so that the matrix $\begin{pmatrix} 2 & 4 \\ -8 & x \end{pmatrix}$ does ***not*** have an inverse ?

(a) 16

(b) -16

(c) 8

(d) -8

21. The system of equations

$2x + y - 3z = 5$,

$3x - 2y + 2z = 5$ and

$5x - 3y - z = 16$

(a) is inconsistent

(b) is consistent, with a unique solution

(c) is consistent, with infinitely many solutions

(d) has its solution lying along x-axis in three-dimensional space

22. Which one of the following is correct in respect of the cube roots of unity ?

(a) They are collinear

(b) They lie on a circle of radius $\sqrt{3}$

(c) They form an equilateral triangle

(d) None of the above

23. If u, v and w (all positive) are the p^{th}, q^{th} and r^{th} terms of a GP, then the determinant of the matrix $\begin{pmatrix} \ln u & p & 1 \\ \ln v & q & 1 \\ \ln w & r & 1 \end{pmatrix}$ is

(a) 0

(b) 1

(c) $(p-q)(q-r)(r-p)$

(d) $\ln u \times \ln v \times \ln w$

24. Let the coefficient of the middle term of the binomial expansion of $(1 + x)^{2n}$ be α and those of two middle terms of the binomial expansion of $(1 + x)^{2n-1}$ be β and γ. Which one of the following relations is correct ?

(a) $\alpha > \beta + \gamma$

(b) $\alpha < \beta + \gamma$

(c) $\alpha = \beta + \gamma$

(d) $\alpha = \beta\gamma$

25. Let $A = \{x \in \mathbf{R} : -1 \leq x \leq 1\}$, $B = \{y \in \mathbf{R} : -1 \leq y \leq 1\}$ and S be the subset of $A \times B$, defined by $S = \{(x, y) \in A \times B : x^2 + y^2 = 1\}$.

Which one of the following is correct ?

(a) S is a one-one function from A into B

(b) S is a many-one function from A into B

(c) S is a bijective mapping from A into B

(d) S is not a function

26. Let T_r be the r^{th} term of an AP for $r = 1, 2, 3, \ldots$. If for some distinct positive integers m and n we have $T_m = 1/n$ and $T_n = 1/m$, then what is T_{mn} equal to ?

(a) $(mn)^{-1}$

(b) $m^{-1} + n^{-1}$

(c) 1

(d) 0

27. Suppose f(x) is such a quadratic expression that it is positive for all real x.

If $g(x) = f(x) + f'(x) + f''(x)$, then for any real x

(a) $g(x) < 0$

(b) $g(x) > 0$

(c) $g(x) = 0$

(d) $g(x) \geq 0$

28. Consider the following in respect of matrices A, B and C of same order :

1. $(A + B + C)' = A' + B' + C'$
2. $(AB)' = A'B'$
3. $(ABC)' = C'B'A'$

where A' is the transpose of the matrix A.

Which of the above are correct ?

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

29. The sum of the binary numbers $(11011)_2$, $(10110110)_2$ and $(10011x0y)_2$ is the binary number $(101101101)_2$. What are the values of x and y ?

(a) $x = 1, y = 1$

(b) $x = 1, y = 0$

(c) $x = 0, y = 1$

(d) $x = 0, y = 0$

30. Let matrix B be the adjoint of a square matrix A, I be the identity matrix of same order as A. If k ($\neq 0$) is the determinant of the matrix A, then what is AB equal to ?

(a) I

(b) kI

(c) k^2I

(d) $(1/k)I$

31. If $(0{\cdot}2)^x = 2$ and $\log_{10} 2 = 0{\cdot}3010$, then what is the value of x to the nearest tenth ?

(a) $-10{\cdot}0$

(b) $-0{\cdot}5$

(c) $-0{\cdot}4$

(d) $-0{\cdot}2$

32. The total number of 5-digit numbers that can be composed of distinct digits from 0 to 9 is

(a) 45360

(b) 30240

(c) 27216

(d) 15120

33. What is the determinant of the matrix $\begin{pmatrix} x & y & y+z \\ z & x & z+x \\ y & z & x+y \end{pmatrix}$?

(a) $(x-y)(y-z)(z-x)$

(b) $(x-y)(y-z)$

(c) $(y-z)(z-x)$

(d) $(z-x)^2(x+y+z)$

34. If A, B and C are the angles of a triangle and

$$\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A+\sin^2 A & \sin B+\sin^2 B & \sin C+\sin^2 C \end{vmatrix} = 0,$$

then which one of the following is correct ?

(a) The triangle ABC is isosceles

(b) The triangle ABC is equilateral

(c) The triangle ABC is scalene

(d) No conclusion can be drawn with regard to the nature of the triangle

35. Consider the following in respect of matrices A and B of same order :

1. $A^2 - B^2 = (A + B)(A - B)$

2. $(A - I)(I + A) = O \Leftrightarrow A^2 = I$

where I is the identity matrix and O is the null matrix.

Which of the above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

36. What is $\dfrac{2\tan\theta}{1+\tan^2\theta}$ equal to ?

(a) $\cos 2\theta$

(b) $\tan 2\theta$

(c) $\sin 2\theta$

(d) $\operatorname{cosec} 2\theta$

37. If $\sec(\theta - \alpha)$, $\sec\theta$ and $\sec(\theta + \alpha)$ are in AP, where $\cos\alpha \neq 1$, then what is the value of $\sin^2\theta + \cos\alpha$?

(a) 0

(b) 1

(c) -1

(d) $\frac{1}{2}$

38. If $A + B + C = 180°$, then what is $\sin 2A - \sin 2B - \sin 2C$ equal to ?

(a) $-4 \sin A \sin B \sin C$

(b) $-4 \cos A \sin B \cos C$

(c) $-4 \cos A \cos B \sin C$

(d) $-4 \sin A \cos B \cos C$

39. A balloon is directly above one end of a bridge. The angle of depression of the other end of the bridge from the balloon is 48°. If the height of the balloon above the bridge is 122 m, then what is the length of the bridge ?

(a) $122 \sin 48°$ m

(b) $122 \tan 42°$ m

(c) $122 \cos 48°$ m

(d) $122 \tan 48°$ m

40. A is an angle in the fourth quadrant. It satisfies the trigonometric equation $3(3 - \tan^2 A - \cot A)^2 = 1$. Which one of the following is a value of A ?

(a) 300°

(b) 315°

(c) 330°

(d) 345°

41. The top of a hill observed from the top and bottom of a building of height h is at angles of elevation $\frac{\pi}{6}$ and $\frac{\pi}{3}$ respectively. What is the height of the hill ?

(a) 2h

(b) $\frac{3h}{2}$

(c) h

(d) $\frac{h}{2}$

42. What is/are the solution(s) of the trigonometric equation $\operatorname{cosec} x + \cot x = \sqrt{3}$, where $0 < x < 2\pi$?

(a) $\frac{5\pi}{3}$ only

(b) $\frac{\pi}{3}$ only

(c) π only

(d) $\pi, \frac{\pi}{3}, \frac{5\pi}{3}$

43. If $\theta = \frac{\pi}{8}$, then what is the value of

$(2\cos\theta + 1)^{10}(2\cos 2\theta - 1)^{10}(2\cos\theta - 1)^{10}$

$(2\cos 4\theta - 1)^{10}$?

(a) 0

(b) 1

(c) 2

(d) 4

44. If $\cos\alpha$ and $\cos\beta$ $(0 < \alpha < \beta < \pi)$ are the roots of the quadratic equation $4x^2 - 3 = 0$, then what is the value of $\sec\alpha \times \sec\beta$?

(a) $-\frac{4}{3}$

(b) $\frac{4}{3}$

(c) $\frac{3}{4}$

(d) $-\frac{3}{4}$

45. Consider the following values of x :

1. 8
2. -4
3. $\frac{1}{6}$
4. $-\frac{1}{4}$

Which of the above values of x is/are the solution(s) of the equation

$\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$?

(a) 3 only

(b) 2 and 3 only

(c) 1 and 4 only

(d) 4 only

46. If the second term of a GP is 2 and the sum of its infinite terms is 8, then the GP is

(a) $8, 2, \frac{1}{2}, \frac{1}{8}, \ldots$

(b) $10, 2, \frac{2}{5}, \frac{2}{25}, \ldots$

(c) $4, 2, 1, \frac{1}{2}, \frac{1}{2^2}, \ldots$

(d) $6, 3, \frac{3}{2}, \frac{3}{4}, \ldots$

47. If a, b, c are in AP or GP or HP, then $\frac{a-b}{b-c}$ is equal to

(a) $\frac{b}{a}$ or 1 or $\frac{b}{c}$

(b) $\frac{c}{a}$ or $\frac{c}{b}$ or 1

(c) 1 or $\frac{a}{b}$ or $\frac{a}{c}$

(d) 1 or $\frac{a}{b}$ or $\frac{c}{a}$

48. What is the sum of all three-digit numbers that can be formed using all the digits 3, 4 and 5, when repetition of digits is ***not*** allowed ?

(a) 2664

(b) 3882

(c) 4044

(d) 4444

49. The ratio of roots of the equations $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$ are equal. If D_1 and D_2 are respective discriminants, then what is $\frac{D_1}{D_2}$ equal to ?

(a) $\frac{a^2}{p^2}$

(b) $\frac{b^2}{q^2}$

(c) $\frac{c^2}{r^2}$

(d) None of the above

50. If $A = \sin^2 \theta + \cos^4 \theta$, then for all real θ, which one of the following is correct ?

(a) $1 \leq A \leq 2$

(b) $\frac{3}{4} \leq A \leq 1$

(c) $\frac{13}{16} \leq A \leq 1$

(d) $\frac{3}{4} \leq A \leq \frac{13}{16}$

51. The equation of a circle whose end points of a diameter are (x_1, y_1) and (x_2, y_2) is

(a) $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = x^2 + y^2$

(b) $(x - x_1)^2 + (y - y_1)^2 = x_2 y_2$

(c) $x^2 + y^2 + 2x_1 x_2 + 2y_1 y_2 = 0$

(d) $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

52. The second degree equation $x^2 + 4y^2 - 2x - 4y + 2 = 0$ represents

(a) A point

(b) An ellipse of semi-major axis 1

(c) An ellipse with eccentricity $\frac{\sqrt{3}}{2}$

(d) None of the above

53. The angle between the two lines $lx + my + n = 0$ and $l'x + m'y + n' = 0$ is given by $\tan^{-1} \theta$. What is θ equal to ?

(a) $\left|\frac{lm' - l'm}{ll' - mm'}\right|$

(b) $\left|\frac{lm' + l'm}{ll' + mm'}\right|$

(c) $\left|\frac{lm' - l'm}{ll' + mm'}\right|$

(d) $\left|\frac{lm' + l'm}{ll' - mm'}\right|$

54. Consider the following statements :

1. The distance between the lines $y = mx + c_1$ and $y = mx + c_2$ is $\frac{|c_1 - c_2|}{\sqrt{1 + m^2}}$.

2. The distance between the lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is $\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$.

3. The distance between the lines $x = c_1$ and $x = c_2$ is $|c_1 - c_2|$.

Which of the above statements are correct ?

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

55. What is the equation of straight line passing through the point of intersection of the lines $\frac{x}{2} + \frac{y}{3} = 1$ and $\frac{x}{3} + \frac{y}{2} = 1$, and parallel to the line $4x + 5y - 6 = 0$?

(a) $20x + 25y - 54 = 0$

(b) $25x + 20y - 54 = 0$

(c) $4x + 5y - 54 = 0$

(d) $4x + 5y - 45 = 0$

56. What is the distance of the point (2, 3, 4) from the plane $3x - 6y + 2z + 11 = 0$?

(a) 1 unit

(b) 2 units

(c) 3 units

(d) 4 units

57. Coordinates of the points O, P, Q and R are respectively (0, 0, 0), (4, 6, 2m), (2, 0, 2n) and (2, 4, 6). Let L, M, N and K be points on the sides OR, OP, PQ and QR respectively such that LMNK is a parallelogram whose two adjacent sides LK and LM are each of length $\sqrt{2}$. What are the values of m and n respectively ?

(a) 6, 2

(b) 1, 3

(c) 3, 1

(d) None of the above

58. The line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is given by

(a) $x + y + z = 6$, $x + 2y - 3z = -4$

(b) $x + 2y - 2z = -1$, $4x + 4y - 5z - 3 = 0$

(c) $3x + 2y - 3z = 0$, $3x - 6y + 3z = -2$

(d) $3x + 2y - 3z = -2$, $3x - 6y + 3z = 0$

59. Consider the following statements :

1. The angle between the planes $2x - y + z = 1$ and $x + y + 2z = 3$ is $\frac{\pi}{3}$.
2. The distance between the planes $6x - 3y + 6z + 2 = 0$ and $2x - y + 2z + 4 = 0$ is $\frac{10}{9}$.

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

60. Consider the following statements :

Statement I : If the line segment joining the points P(m, n) and Q(r, s) subtends an angle α at the origin, then $\cos\alpha = \frac{ms - nr}{\sqrt{(m^2 + n^2)(r^2 + s^2)}}$.

Statement II : In any triangle ABC, it is true that $a^2 = b^2 + c^2 - 2bc\cos A$.

Which one of the following is correct in respect of the above two statements ?

(a) Both Statement I and Statement II are true and Statement II is the correct explanation of Statement I

(b) Both Statement I and Statement II are true, but Statement II is not the correct explanation of Statement I

(c) Statement I is true, but Statement II is false

(d) Statement I is false, but Statement II is true

61. What is the area of the triangle with vertices $\left(x_1, \frac{1}{x_1}\right), \left(x_2, \frac{1}{x_2}\right), \left(x_3, \frac{1}{x_3}\right)$?

(a) $|(x_1 - x_2)(x_2 - x_3)(x_3 - x_1)|$

(b) 0

(c) $\left|\frac{(x_1 - x_2)(x_2 - x_3)(x_3 - x_1)}{x_1 x_2 x_3}\right|$

(d) $\left|\frac{(x_1 - x_2)(x_2 - x_3)(x_3 - x_1)}{2x_1 x_2 x_3}\right|$

62. If y-axis touches the circle $x^2 + y^2 + gx + fy + \frac{c}{4} = 0$, then the normal at this point intersects the circle at the point

(a) $\left(-\frac{g}{2}, -\frac{f}{2}\right)$

(b) $\left(-g, -\frac{f}{2}\right)$

(c) $\left(-\frac{g}{2}, f\right)$

(d) $(-g, -f)$

63. Let $|\vec{a}| \neq 0$, $|\vec{b}| \neq 0$.

$(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = |\vec{a}|^2 + |\vec{b}|^2$ holds if and only if

(a) $\vec{a}$ and $\vec{b}$ are perpendicular

(b) $\vec{a}$ and $\vec{b}$ are parallel

(c) $\vec{a}$ and $\vec{b}$ are inclined at an angle of 45°

(d) $\vec{a}$ and $\vec{b}$ are anti-parallel

64. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then what is $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k})$ equal to ?

(a) x

(b) $x + y$

(c) $-(x + y + z)$

(d) $(x + y + z)$

65. A unit vector perpendicular to each of the vectors $2\hat{i} - \hat{j} + \hat{k}$ and $3\hat{i} - 4\hat{j} - \hat{k}$ is

(a) $\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$

(b) $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{2}\hat{j} + \frac{1}{2}\hat{k}$

(c) $\frac{1}{\sqrt{3}}\hat{i} - \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$

(d) $\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$

66. If $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{a} - \vec{b}| = 5$, then what is the value of $|\vec{a} + \vec{b}|$?

(a) 8

(b) 6

(c) $5\sqrt{2}$

(d) 5

67. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three mutually perpendicular vectors each of unit magnitude. If $\vec{A} = \vec{a} + \vec{b} + \vec{c}$, $\vec{B} = \vec{a} - \vec{b} + \vec{c}$ and $\vec{C} = \vec{a} - \vec{b} - \vec{c}$, then which one of the following is correct ?

(a) $|\vec{A}| > |\vec{B}| > |\vec{C}|$

(b) $|\vec{A}| = |\vec{B}| \neq |\vec{C}|$

(c) $|\vec{A}| = |\vec{B}| = |\vec{C}|$

(d) $|\vec{A}| \neq |\vec{B}| \neq |\vec{C}|$

68. What is $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})$ equal to ?

(a) $\vec{0}$

(b) $\vec{a} \times \vec{b}$

(c) $2(\vec{a} \times \vec{b})$

(d) $|\vec{a}|^2 - |\vec{b}|^2$

69. A spacecraft located at $\hat{i} + 2\hat{j} + 3\hat{k}$ is subjected to a force $\lambda\hat{k}$ by firing a rocket. The spacecraft is subjected to a moment of magnitude

(a) λ

(b) $\sqrt{3}\lambda$

(c) $\sqrt{5}\lambda$

(d) None of the above

70. In a triangle ABC, if taken in order, consider the following statements :

1. $\vec{AB} + \vec{BC} + \vec{CA} = \vec{0}$
2. $\vec{AB} + \vec{BC} - \vec{CA} = \vec{0}$
3. $\vec{AB} - \vec{BC} + \vec{CA} = \vec{0}$
4. $\vec{BA} - \vec{BC} + \vec{CA} = \vec{0}$

How many of the above statements are correct ?

(a) One

(b) Two

(c) Three

(d) Four

71. Let the slope of the curve $y = \cos^{-1}(\sin x)$ be $\tan\theta$. Then the value of θ in the interval $(0, \pi)$ is

(a) $\frac{\pi}{6}$

(b) $\frac{3\pi}{4}$

(c) $\frac{\pi}{4}$

(d) $\frac{\pi}{2}$

72. If $f(x) = \frac{\sqrt{x-1}}{x-4}$ defines a function on $\mathbf{R}$, then what is its domain ?

(a) $(-\infty, 4) \cup (4, \infty)$

(b) $[4, \infty)$

(c) $(1, 4) \cup (4, \infty)$

(d) $[1, 4) \cup (4, \infty)$

73. Consider the function

$$f(x) = \begin{cases} \frac{\sin 2x}{5x} & \text{if} \quad x \neq 0 \\ \frac{2}{15} & \text{if} \quad x = 0 \end{cases} ;$$

Which one of the following is correct in respect of the function ?

(a) It is not continuous at $x = 0$

(b) It is continuous at every x

(c) It is not continuous at $x = \pi$

(d) It is continuous at $x = 0$

74. For the function $f(x) = |x - 3|$, which one of the following is ***not*** correct ?

(a) The function is not continuous at $x = -3$

(b) The function is continuous at $x = 3$

(c) The function is differentiable at $x = 0$

(d) The function is differentiable at $x = -3$

75. If the function $f(x) = \frac{2x - \sin^{-1} x}{2x + \tan^{-1} x}$ is continuous at each point in its domain, then what is the value of $f(0)$?

(a) $-\frac{1}{3}$

(b) $\frac{1}{3}$

(c) $\frac{2}{3}$

(d) 2

76. If $f(x) = \sqrt{25 - x^2}$, then what is $\lim_{x \to 1} \frac{f(x) - f(1)}{x - 1}$ equal to ?

(a) $-\frac{1}{\sqrt{24}}$

(b) $\frac{1}{\sqrt{24}}$

(c) $-\frac{1}{4\sqrt{3}}$

(d) $\frac{1}{4\sqrt{3}}$

77. If $y = \tan^{-1}\left(\frac{5 - 2\tan\sqrt{x}}{2 + 5\tan\sqrt{x}}\right)$, then what is $\frac{dy}{dx}$ equal to ?

(a) $-\frac{1}{2\sqrt{x}}$

(b) 1

(c) -1

(d) $\frac{1}{2\sqrt{x}}$

78. Which one of the following is correct in respect of the function

$f(x) = x \sin x + \cos x + \frac{1}{2}\cos^2 x$?

(a) It is increasing in the interval $\left(0, \frac{\pi}{2}\right)$

(b) It remains constant in the interval $\left(0, \frac{\pi}{2}\right)$

(c) It is decreasing in the interval $\left(0, \frac{\pi}{2}\right)$

(d) It is decreasing in the interval $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

79. What is $\lim_{\theta \to 0} \frac{\sqrt{1 - \cos\theta}}{\theta}$ equal to ?

(a) $\sqrt{2}$

(b) $2\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$

(d) $-\frac{1}{2\sqrt{2}}$

80. A function $f : A \to \mathbf{R}$ is defined by the equation $f(x) = x^2 - 4x + 5$ where $A = (1, 4)$. What is the range of the function ?

(a) (2, 5)

(b) (1, 5)

(c) [1, 5)

(d) [1, 5]

81. What is $\int_a^b [x]\,dx + \int_a^b [-x]\,dx$ equal to, where [.] is the greatest integer function ?

(a) $b - a$

(b) $a - b$

(c) 0

(d) $2(b - a)$

82. What is $\int_2^8 |x-5|\,dx$ equal to ?

(a) 2

(b) 3

(c) 4

(d) 9

83. What is $\int \sin^3 x \cos x\,dx$ equal to ?

(a) $\cos^4 x + c$

(b) $\sin^4 x + c$

(c) $\frac{(1-\sin^2 x)^2}{4} + c$

(d) $\frac{(1-\cos^2 x)^2}{4} + c$

where c is the constant of integration.

84. What is $\int e^{ln(\tan x)}\,dx$ equal to ?

(a) $ln\,|\tan x| + c$

(b) $ln\,|\sec x| + c$

(c) $\tan x + c$

(d) $e^{\tan x} + c$

where c is the constant of integration.

85. What is $\int_{-1}^{1} \left\{\frac{d}{dx}\left(\tan^{-1}\frac{1}{x}\right)\right\} dx$ equal to ?

(a) 0

(b) $-\frac{\pi}{4}$

(c) $-\frac{\pi}{2}$

(d) $\frac{\pi}{2}$

86. In which one of the following intervals is the function $f(x) = x^2 - 5x + 6$ decreasing ?

(a) $(-\infty, 2]$

(b) $[3, \infty)$

(c) $(-\infty, \infty)$

(d) $(2, 3)$

87. The differential equation of the family of curves $y = p\cos(ax) + q\sin(ax)$, where p, q are arbitrary constants, is

(a) $\frac{d^2y}{dx^2} - a^2y = 0$

(b) $\frac{d^2y}{dx^2} - ay = 0$

(c) $\frac{d^2y}{dx^2} + ay = 0$

(d) $\frac{d^2y}{dx^2} + a^2y = 0$

88. The equation of the curve passing through the point $(-1, -2)$ which satisfies $\frac{dy}{dx} = -x^2 - \frac{1}{x^3}$, is

(a) $17x^2y - 6x^2 + 3x^5 - 2 = 0$

(b) $6x^2y + 17x^2 + 2x^5 - 3 = 0$

(c) $6xy - 2x^2 + 17x^5 + 3 = 0$

(d) $17x^2y + 6xy - 3x^5 + 5 = 0$

89. What is the order of the differential equation whose solution is $y = a \cos x + b \sin x + ce^{-x} + d$, where a, b, c and d are arbitrary constants ?

(a) 1

(b) 2

(c) 3

(d) 4

90. What is the solution of the differential equation $ln\left(\frac{dy}{dx}\right) = ax + by$?

(a) $a e^{ax} + b e^{by} = c$

(b) $\frac{1}{a} e^{ax} + \frac{1}{b} e^{by} = c$

(c) $a e^{ax} + b e^{-by} = c$

(d) $\frac{1}{a} e^{ax} + \frac{1}{b} e^{-by} = c$

where c is an arbitrary constant.

91. If $u = e^{ax} \sin bx$ and $v = e^{ax} \cos bx$, then what is $u\frac{du}{dx} + v\frac{dv}{dx}$ equal to ?

(a) $a e^{2ax}$

(b) $(a^2 + b^2) e^{ax}$

(c) $ab e^{2ax}$

(d) $(a + b) e^{ax}$

92. If $y = \sin (ln\ x)$, then which one of the following is correct ?

(a) $\frac{d^2y}{dx^2} + y = 0$

(b) $\frac{d^2y}{dx^2} = 0$

(c) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

(d) $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$

93. A flower-bed in the form of a sector has been fenced by a wire of 40 m length. If the flower-bed has the greatest possible area, then what is the radius of the sector ?

(a) 25 m

(b) 20 m

(c) 10 m

(d) 5 m

94. What is the minimum value of $[x(x-1)+1]^{\frac{1}{3}}$, where $0 \le x \le 1$?

(a) $\left(\frac{3}{4}\right)^{\frac{1}{3}}$

(b) 1

(c) $\frac{1}{2}$

(d) $\left(\frac{3}{8}\right)^{\frac{1}{3}}$

95. If $y = |\sin x|^{|x|}$, then what is the value of $\frac{dy}{dx}$ at $x = -\frac{\pi}{6}$?

(a) $\frac{2^{-\frac{\pi}{6}}(6\,ln\,2 - \sqrt{3}\pi)}{6}$

(b) $\frac{2^{\frac{\pi}{6}}(6\,ln\,2 + \sqrt{3}\pi)}{6}$

(c) $\frac{2^{-\frac{\pi}{6}}(6\,ln\,2 + \sqrt{3}\pi)}{6}$

(d) $\frac{2^{\frac{\pi}{6}}(6\,ln\,2 - \sqrt{3}\pi)}{6}$

96. What is $\frac{d\sqrt{1-\sin 2x}}{dx}$ equal to, where $\frac{\pi}{4} < x < \frac{\pi}{2}$?

(a) $\cos x + \sin x$

(b) $-(\cos x + \sin x)$

(c) $\pm(\cos x + \sin x)$

(d) None of the above

97. What is $\int \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x}$ equal to ?

(a) $c + \frac{1}{ab}\tan^{-1}\left(\frac{a \tan x}{b}\right)$

(b) $c - \frac{1}{ab}\tan^{-1}\left(\frac{b \tan x}{a}\right)$

(c) $c + \frac{1}{ab}\tan^{-1}\left(\frac{b \tan x}{a}\right)$

(d) None of the above

where c is the constant of integration.

98. Let $f(x + y) = f(x)f(y)$ and $f(x) = 1 + xg(x)\phi(x)$, where $\lim_{x \to 0} g(x) = a$ and $\lim_{x \to 0} \phi(x) = b$. Then what is $f'(x)$ equal to ?

(a) $1 + abf(x)$

(b) $1 + ab$

(c) ab

(d) $abf(x)$

99. What is the solution of the differential equation $\frac{dx}{dy} = \frac{x+y+1}{x+y-1}$?

(a) $y - x + 4\,ln(x + y) = c$

(b) $y + x + 2\,ln(x + y) = c$

(c) $y - x + ln(x + y) = c$

(d) $y + x + 2\,ln(x + y) = c$

where c is an arbitrary constant.

100. What is $\lim_{x \to \frac{\pi}{6}} \frac{2\sin^2 x + \sin x - 1}{2\sin^2 x - 3\sin x + 1}$ equal to ?

(a) $-\frac{1}{2}$

(b) $-\frac{1}{3}$

(c) -2

(d) -3

101. If two dice are thrown and at least one of the dice shows 5, then the probability that the sum is 10 or more is

(a) $\frac{1}{6}$

(b) $\frac{4}{11}$

(c) $\frac{3}{11}$

(d) $\frac{2}{11}$

102. The correlation coefficient computed from a set of 30 observations is 0·8. Then the percentage of variation not explained by linear regression is

(a) 80%

(b) 20%

(c) 64%

(d) 36%

103. The average age of a combined group of men and women is 25 years. If the average age of the group of men is 26 years and that of the group of women is 21 years, then the percentage of men and women in the group is respectively

(a) 20, 80

(b) 40, 60

(c) 60, 40

(d) 80, 20

104. If sin β is the harmonic mean of sin α and cos α, and sin θ is the arithmetic mean of sin α and cos α, then which of the following is/are correct ?

1. $\sqrt{2}\ \sin\left(\alpha + \frac{\pi}{4}\right) \sin\beta = \sin 2\alpha$

2. $\sqrt{2}\ \sin\theta = \cos\left(\alpha - \frac{\pi}{4}\right)$

Select the correct answer using the code given below :

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

105. Let A, B and C be three mutually exclusive and exhaustive events associated with a random experiment. If P(B) = 1·5 P(A) and P(C) = 0·5 P(B), then P(A) is equal to

(a) $\frac{3}{4}$

(b) $\frac{4}{13}$

(c) $\frac{2}{3}$

(d) $\frac{1}{2}$

106. In a bolt factory, machines X, Y, Z manufacture bolts that are respectively 25%, 35% and 40% of the factory's total output. The machines X, Y, Z respectively produce 2%, 4% and 5% defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine X ?

(a) $\frac{5}{39}$

(b) $\frac{14}{39}$

(c) $\frac{20}{39}$

(d) $\frac{34}{39}$

107. 8 coins are tossed simultaneously. The probability of getting at least 6 heads is

(a) $\frac{7}{64}$

(b) $\frac{57}{64}$

(c) $\frac{37}{256}$

(d) $\frac{229}{256}$

108. Three groups of children contain 3 girls and 1 boy; 2 girls and 2 boys; 1 girl and 3 boys. One child is selected at random from each group. The probability that the three selected consist of 1 girl and 2 boys is

(a) $\frac{13}{32}$

(b) $\frac{9}{32}$

(c) $\frac{3}{32}$

(d) $\frac{1}{32}$

109. Consider the following statements :

1. If 10 is added to each entry on a list, then the average increases by 10.
2. If 10 is added to each entry on a list, then the standard deviation increases by 10.
3. If each entry on a list is doubled, then the average doubles.

Which of the above statements are correct ?

(a) 1, 2 and 3

(b) 1 and 2 only

(c) 1 and 3 only

(d) 2 and 3 only

110. The variance of 25 observations is 4. If 2 is added to each observation, then the new variance of the resulting observations is

(a) 2

(b) 4

(c) 6

(d) 8

111. If $x_i > 0$, $y_i > 0$ $(i = 1, 2, 3, \ldots n)$ are the values of two variables X and Y with geometric means P and Q respectively, then the geometric mean of $\frac{X}{Y}$ is

(a) $\frac{P}{Q}$

(b) $\text{antilog}\left(\frac{P}{Q}\right)$

(c) $n(\log P - \log Q)$

(d) $n(\log P + \log Q)$

112. If the probability of simultaneous occurrence of two events A and B is p and the probability that exactly one of A, B occurs is q, then which of the following is/are correct ?

1. $P(\bar{A}) + P(\bar{B}) = 2 - 2p - q$
2. $P(\bar{A} \cap \bar{B}) = 1 - p - q$

Select the correct answer using the code given below :

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

113. If the regression coefficient of Y on X is -6, and the correlation coefficient between X and Y is $-\frac{1}{2}$, then the regression coefficient of X on Y would be

(a) $\frac{1}{24}$

(b) $-\frac{1}{24}$

(c) $-\frac{1}{6}$

(d) $\frac{1}{6}$

114. The set of bivariate observations (x_1, y_1), (x_2, y_2), ..., (x_n, y_n) are such that all the values are distinct and all the observations fall on a straight line with non-zero slope. Then the possible values of the correlation coefficient between x and y are

(a) 0 and 1 only

(b) 0 and -1 only

(c) 0, 1 and -1

(d) -1 and 1 only

115. Two integers x and y are chosen with replacement from the set {0, 1, 2, ..., 10}. The probability that $|x - y| > 5$ is

(a) $\frac{6}{11}$

(b) $\frac{35}{121}$

(c) $\frac{30}{121}$

(d) $\frac{25}{121}$

116. An analysis of monthly wages paid to the workers in two firms A and B belonging to the same industry gives the following result :

	Firm A	Firm B
Number of workers	500	600
Average monthly wage	₹ 1860	₹ 1750
Variance of distribution of wages	81	100

The average of monthly wage and variance of distribution of wages of all the workers in the firms A and B taken together are

(a) ₹ 1860, 100

(b) ₹ 1750, 100

(c) ₹ 1800, 81

(d) None of the above

117. Three dice having digits 1, 2, 3, 4, 5 and 6 on their faces are marked I, II and III and rolled. Let x, y and z represent the number on die-I, die-II and die-III respectively. What is the number of possible outcomes such that $x > y > z$?

(a) 14

(b) 16

(c) 18

(d) 20

118. Which one of the following can be obtained from an ogive ?

(a) Mean

(b) Median

(c) Geometric mean

(d) Mode

119. In any discrete series (when all values are not same), if x represents mean deviation about mean and y represents standard deviation, then which one of the following is correct ?

(a) $y \geq x$

(b) $y \leq x$

(c) $x = y$

(d) $x < y$

120. In which one of the following cases would you expect to get a negative correlation ?

(a) The ages of husbands and wives

(b) Shoe size and intelligence

(c) Insurance companies' profits and the number of claims they have to pay

(d) Amount of rainfall and yield of crop

कच्चे काम के लिए जगह
SPACE FOR ROUGH WORK

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