जब तक आपको यह परीक्षण पुस्तिका खोलने को न कहा जाए तब तक न खोलें
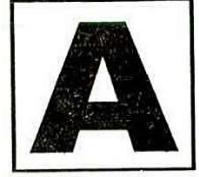
टी.बी.सी. : **ADU–S–LET**

परीक्षण पुस्तिका अनुक्रम

क्रम संख्या   0283425

परीक्षण पुस्तिका
गणित

A

समय : दो घण्टे और तीस मिनट

पूर्णांक : 300

अ नु दे श

1. परीक्षा प्रारम्भ होने के तुरन्त बाद, आप इस परीक्षण पुस्तिका की पड़ताल अवश्य कर लें कि इसमें कोई बिना छपा, फटा या छूटा हुआ पृष्ठ अथवा प्रश्नांश आदि न हो। यदि ऐसा है, तो इसे सही परीक्षण पुस्तिका से बदल लें।

2. कृपया ध्यान रखें कि **OMR** उत्तर-पत्रक में, उचित स्थान पर, रोल नम्बर और परीक्षण पुस्तिका अनुक्रम **A, B, C** या **D** को, ध्यान से एवं बिना किसी चूक या विसंगति के भरने और कूटबद्ध करने की जिम्मेदारी उम्मीदवार की है। किसी भी प्रकार की चूक/विसंगति की स्थिति में उत्तर-पत्रक निरस्त कर दिया जाएगा।

3. इस परीक्षण पुस्तिका पर साथ में दिए गए कोष्ठक में आपको अपना अनुक्रमांक लिखना है। परीक्षण पुस्तिका पर **और कुछ न लिखें।**

4. इस परीक्षण पुस्तिका में **120 प्रश्नांश (प्रश्न)** दिए गए हैं। प्रत्येक प्रश्नांश **हिन्दी** और **अंग्रेजी** दोनों में छपा है। प्रत्येक प्रश्नांश में चार प्रत्युत्तर (उत्तर) दिए गए हैं। इनमें से एक प्रत्युत्तर को चुन लें, जिसे आप उत्तर-पत्रक पर अंकित करना चाहते हैं। यदि आपको ऐसा लगे कि एक से अधिक प्रत्युत्तर सही हैं, तो उस प्रत्युत्तर को अंकित करें जो आपको सर्वोत्तम लगे। प्रत्येक प्रश्नांश के लिए **केवल एक ही** प्रत्युत्तर चुनना है।

5. आपको अपने सभी प्रत्युत्तर अलग से दिए गए उत्तर-पत्रक पर ही अंकित करने हैं। उत्तर-पत्रक में दिए गए निर्देश देखें।

6. **सभी प्रश्नांशों के अंक समान हैं।**

7. इससे पहले कि आप परीक्षण पुस्तिका के विभिन्न प्रश्नांशों के प्रत्युत्तर उत्तर-पत्रक पर अंकित करना शुरू करें, आपको प्रवेश प्रमाण-पत्र के साथ प्रेषित अनुदेशों के अनुसार कुछ विवरण उत्तर-पत्रक में देने हैं।

8. आप अपने सभी प्रत्युत्तरों को उत्तर-पत्रक में भरने के बाद तथा परीक्षा के समापन पर **केवल उत्तर-पत्रक** अधीक्षक को सौंप दें। आपको अपने साथ परीक्षण पुस्तिका ले जाने की अनुमति है।

9. कच्चे काम के लिए पत्रक परीक्षण पुस्तिका के अन्त में संलग्न हैं।

10. गलत उत्तरों के लिए दण्ड :
   वस्तुनिष्ठ प्रश्न-पत्रों में उम्मीदवार द्वारा दिए गए गलत उत्तरों के लिए दण्ड दिया जाएगा।
   (i) प्रत्येक प्रश्न के लिए चार वैकल्पिक उत्तर हैं। उम्मीदवार द्वारा प्रत्येक प्रश्न के लिए दिए गए एक गलत उत्तर के लिए प्रश्न हेतु नियत किए गए अंकों का **एक-तिहाई** दण्ड के रूप में काटा जाएगा।
   (ii) यदि कोई उम्मीदवार एक से अधिक उत्तर देता है, तो इसे **गलत उत्तर** माना जाएगा, यद्यपि दिए गए उत्तरों में से एक उत्तर सही होता है, फिर भी उस प्रश्न के लिए उपर्युक्तानुसार ही उसी तरह का दण्ड दिया जाएगा।
   (iii) यदि उम्मीदवार द्वारा कोई प्रश्न हल नहीं किया जाता है, अर्थात् उम्मीदवार द्वारा उत्तर नहीं दिया जाता है, तो उस प्रश्न के लिए **कोई दण्ड नहीं** दिया जाएगा।

जब तक आपको यह परीक्षण पुस्तिका खोलने को न कहा जाए तब तक न खोलें

Note : **English version of the instructions is printed on the back cover of this Booklet.**

1. If

$$x + \log_{10}(1 + 2^x) = x \log_{10} 5 + \log_{10} 6$$

then x is equal to

(a) 2, –3

(b) 2 only

(c) 1

(d) 3

2. The remainder and the quotient of the binary division $(101110)_2 \div (110)_2$ are respectively

(a) $(111)_2$ and $(100)_2$

(b) $(100)_2$ and $(111)_2$

(c) $(101)_2$ and $(101)_2$

(d) $(100)_2$ and $(100)_2$

3. The matrix A has x rows and $x + 5$ columns. The matrix B has y rows and $11 - y$ columns. Both AB and BA exist. What are the values of x and y respectively?

(a) 8 and 3

(b) 3 and 4

(c) 3 and 8

(d) 8 and 8

4. If $S_n = nP + \dfrac{n(n-1)Q}{2}$, where S_n denotes the sum of the first n terms of an AP, then the common difference is

(a) $P + Q$

(b) $2P + 3Q$

(c) $2Q$

(d) Q

5. The roots of the equation

$$(q - r)x^2 + (r - p)x + (p - q) = 0$$

are

(a) $(r - p)/(q - r)$, $1/2$

(b) $(p - q)/(q - r)$, 1

(c) $(q - r)/(p - q)$, 1

(d) $(r - p)/(p - q)$, $1/2$

6. If E is the universal set and $A = B \cup C$, then the set $E - (E - (E - (E - (E - A))))$ is same as the set

(a) $B' \cup C'$

(b) $B \cup C$

(c) $B' \cap C'$

(d) $B \cap C$

7. If $A = \{x : x$ is a multiple of 2$\}$, $B = \{x : x$ is a multiple of 5$\}$ and $C = \{x : x$ is a multiple of 10$\}$, then $A \cap (B \cap C)$ is equal to

(a) A

(b) B

(c) C

(d) $\{x : x$ is a multiple of 100$\}$

8. If α and β are the roots of the equation $1 + x + x^2 = 0$, then the matrix product

$$\begin{bmatrix} 1 & \beta \\ \alpha & \alpha \end{bmatrix} \begin{bmatrix} \alpha & \beta \\ 1 & \beta \end{bmatrix}$$

is equal to

(a) $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & -1 \\ -1 & -2 \end{bmatrix}$

9. If $|a|$ denotes the absolute value of an integer, then which of the following are correct?

1. $|ab| = |a||b|$

2. $|a + b| \le |a| + |b|$

3. $|a - b| \ge ||a| - |b||$

Select the correct answer using the code given below.

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

10. How many different permutations can be made out of the letters of the word 'PERMUTATION'?

(a) 19958400

(b) 19954800

(c) 19952400

(d) 39916800

11. If $A = \begin{bmatrix} 4i - 6 & 10i \\ 14i & 6 + 4i \end{bmatrix}$ and $k = \dfrac{1}{2i}$, where $i = \sqrt{-1}$, then kA is equal to

(a) $\begin{bmatrix} 2 + 3i & 5 \\ 7 & 2 - 3i \end{bmatrix}$

(b) $\begin{bmatrix} 2 - 3i & 5 \\ 7 & 2 + 3i \end{bmatrix}$

(c) $\begin{bmatrix} 2 - 3i & 7 \\ 5 & 2 + 3i \end{bmatrix}$

(d) $\begin{bmatrix} 2 + 3i & 5 \\ 7 & 2 + 3i \end{bmatrix}$

12. The sum of all real roots of the equation $|x - 3|^2 + |x - 3| - 2 = 0$ is

(a) 2

(b) 3

(c) 4

(d) 6

13. It is given that the roots of the equation $x^2 - 4x - \log_3 P = 0$ are real. For this, the minimum value of P is

(a) $\dfrac{1}{27}$

(b) $\dfrac{1}{64}$

(c) $\dfrac{1}{81}$

(d) 1

14. If A is a square matrix, then the value of $\operatorname{adj} A^T - (\operatorname{adj} A)^T$ is equal to

(a) A

(b) $2|A|I$, where I is the identity matrix

(c) null matrix whose order is same as that of A

(d) unit matrix whose order is same as that of A

15. The value of the product

$6^{\frac{1}{2}} \times 6^{\frac{1}{4}} \times 6^{\frac{1}{8}} \times 6^{\frac{1}{16}} \times \ldots$ up to infinite terms

is

(a) 6

(b) 36

(c) 216

(d) 512

16. The value of the determinant

$$\begin{vmatrix} \cos^2 \dfrac{\theta}{2} & \sin^2 \dfrac{\theta}{2} \\ \sin^2 \dfrac{\theta}{2} & \cos^2 \dfrac{\theta}{2} \end{vmatrix}$$

for all values of θ, is

(a) 1

(b) $\cos \theta$

(c) $\sin \theta$

(d) $\cos 2\theta$

17. The number of terms in the expansion of $(x + a)^{100} + (x - a)^{100}$ after simplification is

(a) 202

(b) 101

(c) 51

(d) 50

18. In the expansion of $(1 + x)^{50}$, the sum of the coefficients of odd powers of x is

(a) 2^{26}

(b) 2^{49}

(c) 2^{50}

(d) 2^{51}

19. If a, b, c are non-zero real numbers, then the inverse of the matrix

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

is equal to

(a) $\begin{bmatrix} a^{-1} & 0 & 0 \\ 0 & b^{-1} & 0 \\ 0 & 0 & c^{-1} \end{bmatrix}$

(b) $\dfrac{1}{abc} \begin{bmatrix} a^{-1} & 0 & 0 \\ 0 & b^{-1} & 0 \\ 0 & 0 & c^{-1} \end{bmatrix}$

(c) $\dfrac{1}{abc} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(d) $\dfrac{1}{abc} \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$

20. A person is to count 4500 notes. Let a_n denote the number of notes he counts in the nth minute. If $a_1 = a_2 = a_3 = \ldots = a_{10} = 150$, and a_{10}, a_{11}, $a_{12}, \ldots$ are in AP with the common difference -2, then the time taken by him to count all the notes is

(a) 24 minutes

(b) 34 minutes

(c) 125 minutes

(d) 135 minutes

21. The smallest positive integer n for which $\left(\dfrac{1+i}{1-i}\right)^n = 1$, is

(a) 1

(b) 4

(c) 8

(d) 16

22. If we define a relation R on the set $N \times N$ as $(a, b) \, R \, (c, d) \Leftrightarrow a + d = b + c$ for all (a, b), $(c, d) \in N \times N$, then the relation is

(a) symmetric only

(b) symmetric and transitive only

(c) equivalence relation

(d) reflexive only

23. If $y = x + x^2 + x^3 + \ldots$ up to infinite terms, where $x < 1$, then which one of the following is correct?

(a) $x = \dfrac{y}{1+y}$

(b) $x = \dfrac{y}{1-y}$

(c) $x = \dfrac{1+y}{y}$

(d) $x = \dfrac{1-y}{y}$

24. If α and β are the roots of the equation $3x^2 + 2x + 1 = 0$, then the equation whose roots are $\alpha + \beta^{-1}$ and $\beta + \alpha^{-1}$ is

(a) $3x^2 + 8x + 16 = 0$

(b) $3x^2 - 8x - 16 = 0$

(c) $3x^2 + 8x - 16 = 0$

(d) $x^2 + 8x + 16 = 0$

25. The value of

$$\frac{1}{\log_3 e} + \frac{1}{\log_3 e^2} + \frac{1}{\log_3 e^4} + ... \text{ up to}$$

infinite terms

is

(a) $\log_e 9$ (b) 0

(c) 1 (d) $\log_e 3$

26. A tea party is arranged for 16 people along two sides of a long table with eight chairs on each side. Four particular men wish to sit on one particular side and two particular men on the other side. The number of ways they can be seated is

(a) $24 \times 8! \times 8!$

(b) $(8!)^3$

(c) $210 \times 8! \times 8!$

(d) $16!$

27. The system of equations $kx + y + z = 1$, $x + ky + z = k$ and $x + y + kz = k^2$ has no solution if k equals

(a) 0 (b) 1

(c) -1 (d) -2

28. If

$$1.3 + 2.3^2 + 3.3^3 + ... + n.3^n = \frac{(2n - 1)3^a + b}{4}$$

then a and b are respectively

(a) $n, 2$

(b) $n, 3$

(c) $n + 1, 2$

(d) $n + 1, 3$

29. In $\triangle PQR$, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of the equation $ax^2 + bx + c = 0$, then which one of the following is correct?

(a) $a = b + c$

(b) $b = c + a$

(c) $c = a + b$

(d) $b = c$

30. If $\left|z - \frac{4}{z}\right| = 2$, then the maximum value of $|z|$ is equal to

(a) $1 + \sqrt{3}$

(b) $1 + \sqrt{5}$

(c) $1 - \sqrt{5}$

(d) $\sqrt{5} - 1$

31. The angle of elevation of a stationary cloud from a point 25 m above a lake is 15° and the angle of depression of its image in the lake is 45°. The height of the cloud above the lake level is

(a) 25 m

(b) $25\sqrt{3}$ m

(c) 50 m

(d) $50\sqrt{3}$ m

32. The value of

$$\tan 9° - \tan 27° - \tan 63° + \tan 81°$$

is equal to

(a) -1

(b) 0

(c) 1

(d) 4

33. The value of $\sqrt{3}\,\mathrm{cosec}\,20° - \sec 20°$ is equal to

(a) 4

(b) 2

(c) 1

(d) -4

34. Angle α is divided into two parts A and B such that $A - B = x$ and $\tan A : \tan B = p : q$. The value of $\sin x$ is equal to

(a) $\dfrac{(p+q)\sin\alpha}{p-q}$

(b) $\dfrac{p\sin\alpha}{p+q}$

(c) $\dfrac{p\sin\alpha}{p-q}$

(d) $\dfrac{(p-q)\sin\alpha}{p+q}$

35. The value of

$$\sin^{-1}\left(\frac{3}{5}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

is equal to

(a) 0

(b) $\dfrac{\pi}{4}$

(c) $\dfrac{\pi}{3}$

(d) $\dfrac{\pi}{2}$

36. The angles of elevation of the top of a tower from the top and foot of a pole are respectively 30° and 45°. If h_T is the height of the tower and h_P is the height of the pole, then which of the following are correct?

1. $\dfrac{2h_P h_T}{3+\sqrt{3}} = h_P^2$

2. $\dfrac{h_T - h_P}{\sqrt{3}+1} = \dfrac{h_P}{2}$

3. $\dfrac{2(h_P + h_T)}{h_P} = 4 + \sqrt{3}$

Select the correct answer using the code given below.

(a) 1 and 3 only

(b) 2 and 3 only

(c) 1 and 2 only

(d) 1, 2 and 3

37. In a triangle ABC, $a - 2b + c = 0$. The value of $\cot\left(\dfrac{A}{2}\right)\cot\left(\dfrac{C}{2}\right)$ is

(a) $\dfrac{9}{2}$

(b) 3

(c) $\dfrac{3}{2}$

(d) 1

38. $\sqrt{1+\sin A} = -\left(\sin\dfrac{A}{2} + \cos\dfrac{A}{2}\right)$ is true if

(a) $\dfrac{3\pi}{2} < A < \dfrac{5\pi}{2}$ only

(b) $\dfrac{\pi}{2} < A < \dfrac{3\pi}{2}$ only

(c) $\dfrac{3\pi}{2} < A < \dfrac{7\pi}{2}$

(d) $0 < A < \dfrac{3\pi}{2}$

39. In triangle ABC, if

$$\frac{\sin^2 A + \sin^2 B + \sin^2 C}{\cos^2 A + \cos^2 B + \cos^2 C} = 2$$

then the triangle is

(a) right-angled

(b) equilateral

(c) isosceles

(d) obtuse-angled

40. The principal value of $\sin^{-1} x$ lies in the interval

(a) $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

(b) $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

(c) $\left[0, \dfrac{\pi}{2}\right]$

(d) $[0, \pi]$

41. The points (a, b), $(0, 0)$, $(-a, -b)$ and (ab, b^2) are

(a) the vertices of a parallelogram

(b) the vertices of a rectangle

(c) the vertices of a square

(d) collinear

42. The length of the normal from origin to the plane $x + 2y - 2z = 9$ is equal to

(a) 2 units

(b) 3 units

(c) 4 units

(d) 5 units

43. If α, β and γ are the angles which the vector $\overrightarrow{OP}$ (O being the origin) makes with positive direction of the coordinate axes, then which of the following are correct?

1. $\cos^2\alpha + \cos^2\beta = \sin^2\gamma$

2. $\sin^2\alpha + \sin^2\beta = \cos^2\gamma$

3. $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

Select the correct answer using the code given below.

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

44. The angle between the lines $x + y - 3 = 0$ and $x - y + 3 = 0$ is α and the acute angle between the lines $x - \sqrt{3}y + 2\sqrt{3} = 0$ and $\sqrt{3}x - y + 1 = 0$ is β. Which one of the following is correct?

(a) $\alpha = \beta$ (b) $\alpha > \beta$

(c) $\alpha < \beta$ (d) $\alpha = 2\beta$

45. Let $\vec{\alpha} = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{\gamma} = 2\hat{i} + \hat{j} + 6\hat{k}$ be three vectors. If $\vec{\alpha}$ and $\vec{\beta}$ are both perpendicular to the vector $\vec{\delta}$ and $\vec{\delta} \cdot \vec{\gamma} = 10$, then what is the magnitude of $\vec{\delta}$?

(a) $\sqrt{3}$ units

(b) $2\sqrt{3}$ units

(c) $\dfrac{\sqrt{3}}{2}$ unit

(d) $\dfrac{1}{\sqrt{3}}$ unit

46. If $\hat{a}$ and $\hat{b}$ are two unit vectors, then the vector $(\hat{a} + \hat{b}) \times (\hat{a} \times \hat{b})$ is parallel to

(a) $(\hat{a} - \hat{b})$

(b) $(\hat{a} + \hat{b})$

(c) $(2\hat{a} - \hat{b})$

(d) $(2\hat{a} + \hat{b})$

47. A force $\vec{F} = \hat{i} + 3\hat{j} + 2\hat{k}$ acts on a particle to displace it from the point $A(\hat{i} + 2\hat{j} - 3\hat{k})$ to the point $B(3\hat{i} - \hat{j} + 5\hat{k})$. The work done by the force will be

(a) 5 units

(b) 7 units

(c) 9 units

(d) 10 units

48. For any vector $\vec{a}$

$$|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2$$

is equal to

(a) $|\vec{a}|^2$

(b) $2|\vec{a}|^2$

(c) $3|\vec{a}|^2$

(d) $4|\vec{a}|^2$

49. A man running round a racecourse notes that the sum of the distances of two flag-posts from him is always 10 m and the distance between the flag-posts is 8 m. The area of the path he encloses is

(a) 18π square metres

(b) 15π square metres

(c) 12π square metres

(d) 8π square metres

50. The distance of the point $(1, 3)$ from the line $2x + 3y = 6$, measured parallel to the line $4x + y = 4$, is

(a) $\dfrac{5}{\sqrt{13}}$ units

(b) $\dfrac{3}{\sqrt{17}}$ unit

(c) $\sqrt{17}$ units

(d) $\dfrac{\sqrt{17}}{2}$ units

51. If the vectors $a\hat{i} + \hat{j} + \hat{k}$, $\hat{i} + b\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + c\hat{k}$ $(a, b, c \neq 1)$ are coplanar, then the value of

$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$$

is equal to

(a) 0

(b) 1

(c) $a + b + c$

(d) abc

52. The point of intersection of the line joining the points $(-3, 4, -8)$ and $(5, -6, 4)$ with the XY-plane is

(a) $\left(\dfrac{7}{3}, -\dfrac{8}{3}, 0\right)$

(b) $\left(-\dfrac{7}{3}, -\dfrac{8}{3}, 0\right)$

(c) $\left(-\dfrac{7}{3}, \dfrac{8}{3}, 0\right)$

(d) $\left(\dfrac{7}{3}, \dfrac{8}{3}, 0\right)$

53. If the angle between the lines whose direction ratios are $\langle 2, -1, 2\rangle$ and $\langle x, 3, 5\rangle$ is $\dfrac{\pi}{4}$, then the smaller value of x is

(a) 52

(b) 4

(c) 2

(d) 1

54. The position of the point $(1, 2)$ relative to the ellipse $2x^2 + 7y^2 = 20$ is

(a) outside the ellipse

(b) inside the ellipse but not at the focus

(c) on the ellipse

(d) at the focus

55. The equation of a straight line which cuts off an intercept of 5 units on negative direction of y-axis and makes an angle $120°$ with positive direction of x-axis is

(a) $y + \sqrt{3}x + 5 = 0$

(b) $y - \sqrt{3}x + 5 = 0$

(c) $y + \sqrt{3}x - 5 = 0$

(d) $y - \sqrt{3}x - 5 = 0$

56. The equation of the line passing through the point $(2, 3)$ and the point of intersection of lines $2x - 3y + 7 = 0$ and $7x + 4y + 2 = 0$ is

(a) $21x + 46y - 180 = 0$

(b) $21x - 46y + 96 = 0$

(c) $46x + 21y - 155 = 0$

(d) $46x - 21y - 29 = 0$

57. The equation of the ellipse whose centre is at origin, major axis is along x-axis with eccentricity $\dfrac{3}{4}$ and latus rectum 4 units is

(a) $\dfrac{x^2}{1024} + \dfrac{7y^2}{64} = 1$

(b) $\dfrac{49x^2}{1024} + \dfrac{7y^2}{64} = 1$

(c) $\dfrac{7x^2}{1024} + \dfrac{49y^2}{64} = 1$

(d) $\dfrac{x^2}{1024} + \dfrac{y^2}{64} = 1$

58. The equation of the circle which passes through the points $(1, 0)$, $(0, -6)$ and $(3, 4)$ is

(a) $4x^2 + 4y^2 + 142x + 47y + 140 = 0$

(b) $4x^2 + 4y^2 - 142x - 47y + 138 = 0$

(c) $4x^2 + 4y^2 - 142x + 47y + 138 = 0$

(d) $4x^2 + 4y^2 + 150x - 49y + 138 = 0$

59. A variable plane passes through a fixed point (a, b, c) and cuts the axes in A, B and C respectively. The locus of the centre of the sphere $OABC$, O being the origin, is

(a) $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$

(b) $\dfrac{a}{x} + \dfrac{b}{y} + \dfrac{c}{z} = 1$

(c) $\dfrac{a}{x} + \dfrac{b}{y} + \dfrac{c}{z} = 2$

(d) $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 2$

60. The equation of the plane passing through the line of intersection of the planes $x + y + z = 1$, $2x + 3y + 4z = 7$, and perpendicular to the plane $x - 5y + 3z = 5$ is given by

(a) $x + 2y + 3z - 6 = 0$

(b) $x + 2y + 3z + 6 = 0$

(c) $3x + 4y + 5z - 8 = 0$

(d) $3x + 4y + 5z + 8 = 0$

61. The inverse of the function $y = 5^{\ln x}$ is

(a) $x = y^{\frac{1}{\ln 5}}$, $y > 0$

(b) $x = y^{\ln 5}$, $y > 0$

(c) $x = y^{\frac{1}{\ln 5}}$, $y < 0$

(d) $x = 5 \ln y$, $y > 0$

62. A function is defined as follows :

$$f(x) = \begin{cases} -\dfrac{x}{\sqrt{x^2}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Which one of the following is correct in respect of the above function?

(a) $f(x)$ is continuous at $x = 0$ but not differentiable at $x = 0$

(b) $f(x)$ is continuous as well as differentiable at $x = 0$

(c) $f(x)$ is discontinuous at $x = 0$

(d) None of the above

63. If $y = (\cos x)^{(\cos x)^{(\cos x)^{\cdot^{\cdot^{\cdot^{\infty}}}}}}$, then $\dfrac{dy}{dx}$ is equal to

(a) $-\dfrac{y^2 \tan x}{1 - y \ln (\cos x)}$

(b) $\dfrac{y^2 \tan x}{1 + y \ln (\cos x)}$

(c) $\dfrac{y^2 \tan x}{1 - y \ln (\sin x)}$

(d) $\dfrac{y^2 \sin x}{1 + y \ln (\sin x)}$

64. Consider the following :

1. $x + x^2$ is continuous at $x = 0$

2. $x + \cos \dfrac{1}{x}$ is discontinuous at $x = 0$

3. $x^2 + \cos \dfrac{1}{x}$ is continuous at $x = 0$

Which of the above are correct?

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

65. Consider the following statements :

1. $\dfrac{dy}{dx}$ at a point on the curve gives slope of the tangent at that point.

2. If $a(t)$ denotes acceleration of a particle, then $\int a(t)\, dt + c$ gives velocity of the particle.

3. If $s(t)$ gives displacement of a particle at time t, then $\dfrac{ds}{dt}$ gives its acceleration at that instant.

Which of the above statements is/are correct?

(a) 1 and 2 only

(b) 2 only

(c) 1 only

(d) 1, 2 and 3

66. If $y = \sec^{-1}\left(\dfrac{x+1}{x-1}\right) + \sin^{-1}\left(\dfrac{x-1}{x+1}\right)$, then $\dfrac{dy}{dx}$ is equal to

(a) 0

(b) 1

(c) $\dfrac{x-1}{x+1}$

(d) $\dfrac{x+1}{x-1}$

67. What is $\int \tan^{-1}(\sec x + \tan x)\,dx$ equal to?

(a) $\dfrac{\pi x}{4} + \dfrac{x^2}{4} + c$

(b) $\dfrac{\pi x}{2} + \dfrac{x^2}{4} + c$

(c) $\dfrac{\pi x}{4} + \dfrac{\pi x^2}{4} + c$

(d) $\dfrac{\pi x}{4} - \dfrac{x^2}{4} + c$

68. A function is defined in $(0, \infty)$ by

$$f(x) = \begin{cases} 1 - x^2 & \text{for } 0 < x \le 1 \\ \ln x & \text{for } 1 < x \le 2 \\ \ln 2 - 1 + 0 \cdot 5x & \text{for } 2 < x < \infty \end{cases}$$

Which one of the following is correct in respect of the derivative of the function, i.e., $f'(x)$?

(a) $f'(x) = 2x$ for $0 < x \le 1$

(b) $f'(x) = -2x$ for $0 < x \le 1$

(c) $f'(x) = -2x$ for $0 < x < 1$

(d) $f'(x) = 0$ for $0 < x < \infty$

69. Which one of the following is correct in respect of the function

$$f(x) = x(x-1)(x+1)?$$

(a) The local maximum value is larger than local minimum value

(b) The local maximum value is smaller than local minimum value

(c) The function has no local maximum

(d) The function has no local minimum

70. Consider the following statements :

1. Derivative of $f(x)$ may not exist at some point.

2. Derivative of $f(x)$ may exist finitely at some point.

3. Derivative of $f(x)$ may be infinite (geometrically) at some point.

Which of the above statements are correct?

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

71. The maximum value of $\dfrac{\ln x}{x}$ is

(a) e

(b) $\dfrac{1}{e}$

(c) $\dfrac{2}{e}$

(d) 1

72. The function $f(x) = |x| - x^3$ is

(a) odd

(b) even

(c) both even and odd

(d) neither even nor odd

73. If

$$l_1 = \frac{d}{dx}(e^{\sin x})$$

$$l_2 = \lim_{h \to 0} \frac{e^{\sin(x+h)} - e^{\sin x}}{h}$$

$$l_3 = \int e^{\sin x} \cos x \, dx$$

then which one of the following is correct?

(a) $l_1 \neq l_2$ (b) $\frac{d}{dx}(l_3) = l_2$

(c) $\int l_3 dx = l_2$ (d) $l_2 = l_3$

74. The general solution of

$$\frac{dy}{dx} = \frac{ax + h}{by + k}$$

represents a circle only when

(a) $a = b = 0$

(b) $a = -b \neq 0$

(c) $a = b \neq 0, \ h = k$

(d) $a = b \neq 0$

75. If $\lim\limits_{x \to \frac{\pi}{2}} \frac{\sin x}{x} = l$ and $\lim\limits_{x \to \infty} \frac{\cos x}{x} = m$, then which one of the following is correct?

(a) $l = 1, \ m = 1$ (b) $l = \frac{2}{\pi}, \ m = \infty$

(c) $l = \frac{2}{\pi}, \ m = 0$ (d) $l = 1, \ m = \infty$

76. What is $\int_0^{2\pi} \sqrt{1 + \sin\frac{x}{2}} \, dx$ equal to?

(a) 8

(b) 4

(c) 2

(d) 0

77. The area bounded by the curve $|x| + |y| = 1$ is

(a) 1 square unit

(b) $2\sqrt{2}$ square units

(c) 2 square units

(d) $2\sqrt{3}$ square units

78. If x is any real number, then $\dfrac{x^2}{1 + x^4}$ belongs to which one of the following intervals?

(a) $(0, 1)$

(b) $\left(0, \frac{1}{2}\right]$

(c) $\left(0, \frac{1}{2}\right)$

(d) $[0, 1]$

79. The left-hand derivative of

$$f(x) = [x] \sin(\pi x) \text{ at } x = k$$

where k is an integer and $[x]$ is the greatest integer function, is

(a) $(-1)^k (k-1)\pi$

(b) $(-1)^{k-1}(k-1)\pi$

(c) $(-1)^k k\pi$

(d) $(-1)^{k-1} k\pi$

80. If $f(x) = \dfrac{x}{2} - 1$, then on the interval $[0, \pi]$ which one of the following is correct?

(a) $\tan[f(x)]$, where $[\cdot]$ is the greatest integer function, and $\dfrac{1}{f(x)}$ are both continuous

(b) $\tan[f(x)]$, where $[\cdot]$ is the greatest integer function, and $f^{-1}(x)$ are both continuous

(c) $\tan[f(x)]$, where $[\cdot]$ is the greatest integer function, and $\dfrac{1}{f(x)}$ are both discontinuous

(d) $\tan[f(x)]$, where $[\cdot]$ is the greatest integer function, is discontinuous but $\dfrac{1}{f(x)}$ is continuous

81. The order and degree of the differential equation

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \rho^2 \left[\frac{d^2 y}{dx^2}\right]^2$$

are respectively

(a) 3 and 2

(b) 2 and 2

(c) 2 and 3

(d) 1 and 3

82. If $y = \cos^{-1}\left(\dfrac{2x}{1+x^2}\right)$, then $\dfrac{dy}{dx}$ is equal to

(a) $-\dfrac{2}{1+x^2}$ for all $|x| < 1$

(b) $-\dfrac{2}{1+x^2}$ for all $|x| > 1$

(c) $\dfrac{2}{1+x^2}$ for all $|x| < 1$

(d) None of the above

83. The set of all points, where the function $f(x) = \sqrt{1 - e^{-x^2}}$ is differentiable, is

(a) $(0, \infty)$

(b) $(-\infty, \infty)$

(c) $(-\infty, 0) \cup (0, \infty)$

(d) $(-1, \infty)$

84. Match List–I with List–II and select the correct answer using the code given below the lists :

List–I (Function)		List–II (Maximum value)
A.	$\sin x + \cos x$	1. $\sqrt{10}$
B.	$3\sin x + 4\cos x$	2. $\sqrt{2}$
C.	$2\sin x + \cos x$	3. 5
D.	$\sin x + 3\cos x$	4. $\sqrt{5}$

Code :

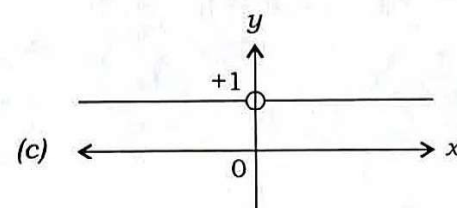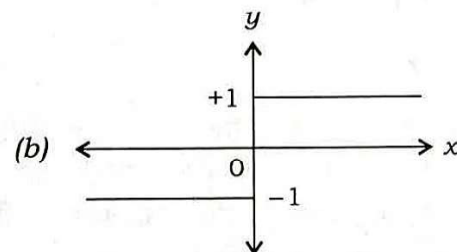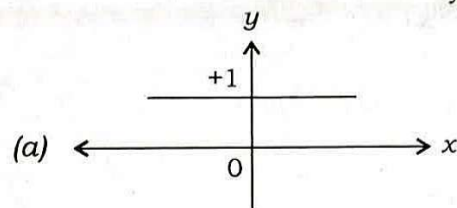|	A	B	C	D
(a)	2	3	1	4
(b)	2	3	4	1
(c)	3	2	1	4
(d)	3	2	4	1

85. If $f(x) = x(\sqrt{x} - \sqrt{x+1})$, then $f(x)$ is

(a) continuous but not differentiable at $x = 0$

(b) differentiable at $x = 0$

(c) not continuous at $x = 0$

(d) None of the above

86. Which one of the following graph represents the function $f(x) = \dfrac{x}{x}$, $x \neq 0$?

(a)

(b)

(c)

(d) None of the above

87. Let $f(n) = \left[\dfrac{1}{4} + \dfrac{n}{1000} \right]$, where $[x]$ denotes the integral part of x. Then the value of $\sum\limits_{n=1}^{1000} f(n)$ is

(a) 251

(b) 250

(c) 1

(d) 0

88. $\int (\ln x)^{-1} dx - \int (\ln x)^{-2} dx$ is equal to

(a) $x (\ln x)^{-1} + c$

(b) $x (\ln x)^{-2} + c$

(c) $x (\ln x) + c$

(d) $x (\ln x)^2 + c$

89. A cylindrical jar without a lid has to be constructed using a given surface area of a metal sheet. If the capacity of the jar is to be maximum, then the diameter of the jar must be k times the height of the jar. The value of k is

(a) 1

(b) 2

(c) 3

(d) 4

90. The value of

$$\int_0^{\frac{\pi}{4}} \sqrt{\tan x}\ dx + \int_0^{\frac{\pi}{4}} \sqrt{\cot x}\ dx$$

is equal to

(a) $\dfrac{\pi}{4}$

(b) $\dfrac{\pi}{2}$

(c) $\dfrac{\pi}{2\sqrt{2}}$

(d) $\dfrac{\pi}{\sqrt{2}}$

91. Let g be the greatest integer function Then the function $f(x) = (g(x))^2 - g(x$ is discontinuous at

(a) all integers

(b) all integers except 0 and 1

(c) all integers except 0

(d) all integers except 1

92. The differential equation of minimu order by eliminating the arbitrar constants A and C in the equatio $y = A[\sin (x + C) + \cos (x + C)]$ is

(a) $y'' + (\sin x + \cos x) y' = 1$

(b) $y'' = (\sin x + \cos x) y'$

(c) $y'' = (y')^2 + \sin x \cos x$

(d) $y'' + y = 0$

93. Consider the following statements :

Statement I :

$x > \sin x$ for all $x > 0$

Statement II :

$f(x) = x - \sin x$ is an increasin function for all $x > 0$

Which one of the following is correct i respect of the above statements?

(a) Both Statement I and Statement are true and Statement II is th correct explanation of Statement

(b) Both Statement I and Statement are true and Statement II is not th correct explanation of Statement

(c) Statement I is true but Statement is false

(d) Statement I is false but Statement is true

94. The solution of the differential equation

$$\frac{dy}{dx} = \frac{y\phi'(x) - y^2}{\phi(x)}$$

is

(a) $y = \dfrac{x}{\phi(x) + c}$

(b) $y = \dfrac{\phi(x)}{x} + c$

(c) $y = \dfrac{\phi(x) + c}{x}$

(d) $y = \dfrac{\phi(x)}{x + c}$

95. If $f(x) = \dfrac{4x + x^4}{1 + 4x^3}$ and $g(x) = \ln\left(\dfrac{1+x}{1-x}\right)$,

then what is the value of $f \circ g\left(\dfrac{e-1}{e+1}\right)$

equal to?

(a) 2

(b) 1

(c) 0

(d) $\dfrac{1}{2}$

96. The value of the determinant

$$\begin{vmatrix} 1-\alpha & \alpha - \alpha^2 & \alpha^2 \\ 1-\beta & \beta - \beta^2 & \beta^2 \\ 1-\gamma & \gamma - \gamma^2 & \gamma^2 \end{vmatrix}$$

is equal to

(a) $(\alpha - \beta)(\beta - \gamma)(\alpha - \gamma)$

(b) $(\alpha - \beta)(\beta - \gamma)(\gamma - \alpha)$

(c) $(\alpha - \beta)(\beta - \gamma)(\gamma - \alpha)(\alpha + \beta + \gamma)$

(d) 0

97. The adjoint of the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

is

(a) $\begin{bmatrix} -1 & 6 & 2 \\ -2 & 1 & -4 \\ 6 & 3 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 6 & -2 \\ -2 & 1 & 4 \\ 6 & -3 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 6 & 1 & 2 \\ 4 & -1 & 2 \\ 6 & 3 & -1 \end{bmatrix}$

(d) $\begin{bmatrix} -6 & 2 & 1 \\ 4 & -2 & 1 \\ 3 & 1 & -6 \end{bmatrix}$

98. If $A = \begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}$, then which one of the

following is correct?

(a) $A^2 = -2A$

(b) $A^2 = -4A$

(c) $A^2 = -3A$

(d) $A^2 = 4A$

99. Geometrically $\mathrm{Re}\,(z^2 - i) = 2$, where $i = \sqrt{-1}$ and Re is the real part, represents

(a) circle

(b) ellipse

(c) rectangular hyperbola

(d) parabola

100. If $p + q + r = a + b + c = 0$, then the determinant

$$\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix}$$

equals

(a) 0

(b) 1

(c) $pa + qb + rc$

(d) $pa + qb + rc + a + b + c$

101. A committee of two persons is selected from two men and two women. The probability that the committee will have exactly one woman is

(a) $\dfrac{1}{6}$

(b) $\dfrac{2}{3}$

(c) $\dfrac{1}{3}$

(d) $\dfrac{1}{2}$

102. Let a die be loaded in such a way that even faces are twice likely to occur as the odd faces. What is the probability that a prime number will show up when the die is tossed?

(a) $\dfrac{1}{3}$

(b) $\dfrac{2}{3}$

(c) $\dfrac{4}{9}$

(d) $\dfrac{5}{9}$

103. Let the sample space consist of non-negative integers up to 50, X denote the numbers which are multiples of 3 and Y denote the odd numbers. Which of the following is/are correct?

1. $P(X) = \dfrac{8}{25}$

2. $P(Y) = \dfrac{1}{2}$

Select the correct answer using the code given below.

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

104. For two events A and B, let $P(A) = \dfrac{1}{2}$, $P(A \cup B) = \dfrac{2}{3}$ and $P(A \cap B) = \dfrac{1}{6}$. What is $P(\overline{A} \cap B)$ equal to?

(a) $\dfrac{1}{6}$

(b) $\dfrac{1}{4}$

(c) $\dfrac{1}{3}$

(d) $\dfrac{1}{2}$

105. Consider the following statements :

1. Coefficient of variation depends on the unit of measurement of the variable.

2. Range is a measure of dispersion.

3. Mean deviation is least when measured about median.

Which of the above statements are correct?

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

106. Given that the arithmetic mean and standard deviation of a sample of 15 observations are 24 and 0 respectively. Then which one of the following is the arithmetic mean of the smallest five observations in the data?

(a) 0

(b) 8

(c) 16

(d) 24

107. Which one of the following can be considered as appropriate pair of values of regression coefficient of y on x and regression coefficient of x on y?

(a) $(1, 1)$

(b) $(-1, 1)$

(c) $\left(-\dfrac{1}{2},\ 2\right)$

(d) $\left(\dfrac{1}{3},\ \dfrac{10}{3}\right)$

108. Let A and B be two events with $P(A) = \dfrac{1}{3}$, $P(B) = \dfrac{1}{6}$ and $P(A \cap B) = \dfrac{1}{12}$. What is $P(B|\overline{A})$ equal to?

(a) $\dfrac{1}{5}$

(b) $\dfrac{1}{7}$

(c) $\dfrac{1}{8}$

(d) $\dfrac{1}{10}$

109. In a binomial distribution, the mean is $\dfrac{2}{3}$ and the variance is $\dfrac{5}{9}$. What is the probability that $X = 2$?

(a) $\dfrac{5}{36}$

(b) $\dfrac{25}{36}$

(c) $\dfrac{25}{216}$

(d) $\dfrac{25}{54}$

110. The probability that a ship safely reaches a port is $\dfrac{1}{3}$. The probability that out of 5 ships, at least 4 ships would arrive safely is

(a) $\dfrac{1}{243}$

(b) $\dfrac{10}{243}$

(c) $\dfrac{11}{243}$

(d) $\dfrac{13}{243}$

111. What is the probability that at least two persons out of a group of three persons were born in the same month (disregard year)?

(a) $\dfrac{33}{144}$

(b) $\dfrac{17}{72}$

(c) $\dfrac{1}{144}$

(d) $\dfrac{2}{9}$

112. It is given that $\overline{X} = 10$, $\overline{Y} = 90$, $\sigma_X = 3$, $\sigma_Y = 12$ and $r_{XY} = 0\cdot8$. The regression equation of X on Y is

(a) $Y = 3\cdot2X + 58$

(b) $X = 3\cdot2Y + 58$

(c) $X = -8 + 0\cdot2Y$

(d) $Y = -8 + 0\cdot2X$

113. If $P(B) = \dfrac{3}{4}$, $P(A \cap B \cap \overline{C}) = \dfrac{1}{3}$ and $P(\overline{A} \cap B \cap \overline{C}) = \dfrac{1}{3}$, then what is $P(B \cap C)$ equal to?

(a) $\dfrac{1}{12}$

(b) $\dfrac{3}{4}$

(c) $\dfrac{1}{15}$

(d) $\dfrac{1}{9}$

114. The following table gives the monthly expenditure of two families :

Items	Expenditure (in ₹)	
	Family A	Family B
Food	3,500	2,700
Clothing	500	800
Rent	1,500	1,000
Education	2,000	1,800
Miscellaneous	2,500	1,800

In constructing a pie diagram to the above data, the radii of the circles are to be chosen by which one of the following ratios?

(a) $1 : 1$

(b) $10 : 9$

(c) $100 : 91$

(d) $5 : 4$

115. If a variable takes values 0, 1, 2, 3, ..., n with frequencies

$$1, C(n, 1), C(n, 2), C(n, 3), ..., C(n, n)$$

respectively, then the arithmetic mean is

(a) $2n$

(b) $n+1$

(c) n

(d) $\dfrac{n}{2}$

116. In a multiple-choice test, an examinee either knows the correct answer with probability p, or guesses with probability $1-p$. The probability of answering a question correctly is $\dfrac{1}{m}$, if he or she merely guesses. If the examinee answers a question correctly, the probability that he or she really knows the answer is

(a) $\dfrac{mp}{1+mp}$

(b) $\dfrac{mp}{1+(m-1)p}$

(c) $\dfrac{(m-1)p}{1+(m-1)p}$

(d) $\dfrac{(m-1)p}{1+mp}$

117. If x_1 and x_2 are positive quantities, then the condition for the difference between the arithmetic mean and the geometric mean to be greater than 1 is

(a) $x_1 + x_2 > 2\sqrt{x_1 x_2}$

(b) $\sqrt{x_1} + \sqrt{x_2} > \sqrt{2}$

(c) $|\sqrt{x_1} - \sqrt{x_2}| > \sqrt{2}$

(d) $x_1 + x_2 < 2(\sqrt{x_1 x_2} + 1)$

118. Consider the following statements :

1. Variance is unaffected by change of origin and change of scale.

2. Coefficient of variance is independent of the unit of observations.

Which of the statements given above is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

119. Five sticks of length 1, 3, 5, 7 and 9 feet are given. Three of these sticks are selected at random. What is the probability that the selected sticks can form a triangle?

(a) 0·5

(b) 0·4

(c) 0·3

(d) 0

120. The coefficient of correlation when coefficients of regression are 0·2 and 1·8 is

(a) 0·36

(b) 0·2

(c) 0·6

(d) 0·9

SPACE FOR ROUGH WORK

SPACE FOR ROUGH WORK

★★★