

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO

T.B.C. : B-OEBA-P-QEX

Test Booklet Series

TEST BOOKLET
MATHEMATICS



Time Allowed : Two Hours and Thirty Minutes

Maximum Marks : 300

INSTRUCTIONS

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET *DOES NOT* HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS, ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.
2. Please note that it is the candidate's responsibility to encode and fill in the Roll Number and Test Booklet Series A, B, C or D carefully and without any omission or discrepancy at the appropriate places in the OMR Answer Sheet. Any omission/discrepancy will render the Answer Sheet liable for rejection.
3. You have to enter your Roll Number on the Test Booklet in the Box provided alongside. *DO NOT* write *anything else* on the Test Booklet. 
4. This Test Booklet contains **120** items (questions). Each item is printed both in **Hindi** and **English**. Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose *ONLY ONE* response for each item.
5. You have to mark all your responses *ONLY* on the separate Answer Sheet provided. See directions in the Answer Sheet.
6. **All** items carry equal marks.
7. Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions sent to you with your Admission Certificate.
8. After you have completed filling in all your responses on the Answer Sheet and the examination has concluded, you should hand over to the Invigilator *only the Answer Sheet*. You are permitted to take away with you the Test Booklet.
9. Sheets for rough work are appended in the Test Booklet at the end.
10. **Penalty for wrong answers :**
THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
 - (i) There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to that question will be deducted as penalty.
 - (ii) If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.
 - (iii) If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

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ध्यान दें : अनुदेशों का हिन्दी रूपान्तर इस पुस्तिका के पिछले पृष्ठ पर छपा है।

1. Suppose ω is a cube root of unity with $\omega \neq 1$. Suppose P and Q are the points on the complex plane defined by ω and ω^2 . If O is the origin, then what is the angle between OP and OQ ?

- (a) 60°
(b) 90°
(c) 120°
(d) 150°

2. Suppose there is a relation $*$ between the positive numbers x and y given by $x*y$ if and only if $x \leq y^2$. Then which one of the following is correct?

- (a) $*$ is reflexive but not transitive and symmetric
(b) $*$ is transitive but not reflexive and symmetric
(c) $*$ is symmetric and reflexive but not transitive
(d) $*$ is symmetric but not reflexive and transitive

3. If $x^2 - px + 4 > 0$ for all real values of x , then which one of the following is correct?

- (a) $|p| < 4$
(b) $|p| \leq 4$
(c) $|p| > 4$
(d) $|p| \geq 4$

4. If $z = x + iy = \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} \right)^{-25}$, where $i = \sqrt{-1}$, then what is the fundamental amplitude of $\frac{z - \sqrt{2}}{z - i\sqrt{2}}$?

- (a) π
(b) $\frac{\pi}{2}$
(c) $\frac{\pi}{3}$
(d) $\frac{\pi}{4}$

5. If

$$f(x_1) - f(x_2) = f\left(\frac{x_1 - x_2}{1 - x_1 x_2}\right)$$

for $x_1, x_2 \in (-1, 1)$, then what is $f(x)$ equal to?

- (a) $\ln\left(\frac{1-x}{1+x}\right)$
(b) $\ln\left(\frac{2+x}{1-x}\right)$
(c) $\tan^{-1}\left(\frac{1-x}{1+x}\right)$
(d) $\tan^{-1}\left(\frac{1+x}{1-x}\right)$

6. What is the range of the function

$$y = \frac{x^2}{1+x^2}$$

where $x \in \mathbb{R}$?

- (a) $[0, 1)$
(b) $[0, 1]$
(c) $(0, 1)$
(d) $(0, 1]$

7. A straight line intersects x and y axes at P and Q respectively. If $(3, 5)$ is the middle point of PQ , then what is the area of the triangle OPQ ?

(a) 12 square units
(b) 15 square units
(c) 20 square units
(d) 30 square units

8. If a circle of radius b units with centre at $(0, b)$ touches the line $y = x - \sqrt{2}$, then what is the value of b ?

(a) $2 + \sqrt{2}$
(b) $2 - \sqrt{2}$
(c) $2\sqrt{2}$
(d) $\sqrt{2}$

For the next three (3) items that follow :

Consider the function

$$f(\theta) = 4(\sin^2 \theta + \cos^4 \theta)$$

9. What is the maximum value of the function $f(\theta)$?

(a) 1
(b) 2
(c) 3
(d) 4

10. What is the minimum value of the function $f(\theta)$?

(a) 0
(b) 1
(c) 2
(d) 3

11. Consider the following statements :

1. $f(\theta) = 2$ has no solution.
2. $f(\theta) = \frac{7}{2}$ has a solution.

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

For the next two (2) items that follow :

Consider the curves

$$f(x) = x|x| - 1 \text{ and } g(x) = \begin{cases} \frac{3x}{2}, & x > 0 \\ 2x, & x \leq 0 \end{cases}$$

12. Where do the curves intersect?

(a) At $(2, 3)$ only

(b) At $(-1, -2)$ only

(c) At $(2, 3)$ and $(-1, -2)$

(d) Neither at $(2, 3)$ nor at $(-1, -2)$

13. What is the area bounded by the curves?

- (a) $\frac{17}{6}$ square units
- (b) $\frac{8}{3}$ square units
- (c) 2 square units
- (d) $\frac{1}{3}$ square unit

For the next two (2) items that follow :

Consider the function

$$f(x) = \frac{27(x^{2/3} - x)}{4}$$

14. How many solutions does the function $f(x) = 1$ have?

- (a) One
- (b) Two
- (c) Three
- (d) Four

15. How many solutions does the function $f(x) = -1$ have?

- (a) One
- (b) Two
- (c) Three
- (d) Four

For the next two (2) items that follow :

Consider the functions

$$f(x) = xg(x) \text{ and } g(x) = \left\lfloor \frac{1}{x} \right\rfloor$$

where $\lfloor \cdot \rfloor$ is the greatest integer function.

16. What is $\int_{\frac{1}{3}}^{\frac{1}{2}} g(x) dx$ equal to?

- (a) $\frac{1}{6}$
- (b) $\frac{1}{3}$
- (c) $\frac{5}{18}$
- (d) $\frac{5}{36}$

17. What is $\int_{\frac{1}{3}}^1 f(x) dx$ equal to?

- (a) $\frac{37}{72}$
- (b) $\frac{2}{3}$
- (c) $\frac{17}{72}$
- (d) $\frac{37}{144}$

For the next five (5) items that follow :

Consider the function

$$f(x) = |x - 1| + x^2$$

where $x \in \mathbb{R}$.

18. Which one of the following statements is correct?

- (a) $f(x)$ is continuous but not differentiable at $x = 0$
- (b) $f(x)$ is continuous but not differentiable at $x = 1$
- (c) $f(x)$ is differentiable at $x = 1$
- (d) $f(x)$ is not differentiable at $x = 0$ and $x = 1$

19. Which one of the following statements is correct?

- (a) $f(x)$ is increasing in $\left(-\infty, \frac{1}{2}\right)$ and decreasing in $\left(\frac{1}{2}, \infty\right)$
- (b) $f(x)$ is decreasing in $\left(-\infty, \frac{1}{2}\right)$ and increasing in $\left(\frac{1}{2}, \infty\right)$
- (c) $f(x)$ is increasing in $(-\infty, 1)$ and decreasing in $(1, \infty)$
- (d) $f(x)$ is decreasing in $(-\infty, 1)$ and increasing in $(1, \infty)$

20. Which one of the following statements is correct?

- (a) $f(x)$ has local minima at more than one point in $(-\infty, \infty)$
- (b) $f(x)$ has local maxima at more than one point in $(-\infty, \infty)$
- (c) $f(x)$ has local minimum at one point only in $(-\infty, \infty)$
- (d) $f(x)$ has neither maxima nor minima in $(-\infty, \infty)$

21. What is the area of the region bounded by x -axis, the curve $y = f(x)$ and the two ordinates $x = \frac{1}{2}$ and $x = 1$?

- (a) $\frac{5}{12}$ square unit
- (b) $\frac{5}{6}$ square unit
- (c) $\frac{7}{6}$ square units
- (d) 2 square units

22. What is the area of the region bounded by x -axis, the curve $y = f(x)$ and the two ordinates $x = 1$ and $x = \frac{3}{2}$?

- (a) $\frac{5}{12}$ square unit
- (b) $\frac{7}{12}$ square unit
- (c) $\frac{2}{3}$ square unit
- (d) $\frac{11}{12}$ square unit

For the next two (2) items that follow :

Given that

$$a_n = \int_0^\pi \frac{\sin^2 \{(n+1)x\}}{\sin 2x} dx$$

23. Consider the following statements :

1. The sequence $\{a_{2n}\}$ is in AP with common difference zero.
2. The sequence $\{a_{2n+1}\}$ is in AP with common difference zero.

Which of the above statements is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

24. What is $a_{n-1} - a_{n-4}$ equal to?

- (a) -1
- (b) 0
- (c) 1
- (d) 2

For the next two (2) items that follow :

Consider the equation $x + |y| = 2y$.

25. Which of the following statements are **not** correct?

1. y as a function of x is not defined for all real x .
2. y as a function of x is not continuous at $x = 0$.
3. y as a function of x is differentiable for all x .

Select the correct answer using the code given below.

- (a) 1 and 2 only
(b) 2 and 3 only
(c) 1 and 3 only
(d) 1, 2 and 3

26. What is the derivative of y as a function of x with respect to x for $x < 0$?

- (a) 2 (b) 1
(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

For the next two (2) items that follow :

Consider the lines

$$y = 3x, y = 6x \text{ and } y = 9$$

27. What is the area of the triangle formed by these lines?

- (a) $\frac{27}{4}$ square units
(b) $\frac{27}{2}$ square units
(c) $\frac{19}{4}$ square units
(d) $\frac{19}{2}$ square units

28. The centroid of the triangle is at which one of the following points?

- (a) (3, 6)
(b) $\left(\frac{3}{2}, 6\right)$
(c) (3, 3)
(d) $\left(\frac{3}{2}, 9\right)$

For the next two (2) items that follow :

Consider the function

$$f(x) = (x - 1)^2 (x + 1)(x - 2)^3$$

29. What is the number of points of local minima of the function $f(x)$?

- (a) None
(b) One
(c) Two
(d) Three

30. What is the number of points of local maxima of the function $f(x)$?

- (a) None
(b) One
(c) Two
(d) Three

31. Let $f(x)$ and $g(x)$ be twice differentiable functions on $[0, 2]$ satisfying $f''(x) = g''(x)$, $f'(1) = 4$, $g'(1) = 6$, $f(2) = 3$ and $g(2) = 9$. Then what is $f(x) - g(x)$ at $x = 4$ equal to?

- (a) -10
(b) -6
(c) -4
(d) 2

For the next two (2) items that follow :

Consider the curves

$$y = |x - 1| \text{ and } |x| = 2$$

32. What is/are the point(s) of intersection of the curves?

- (a) $(-2, 3)$ only
(b) $(2, 1)$ only
(c) $(-2, 3)$ and $(2, 1)$
(d) Neither $(-2, 3)$ nor $(2, 1)$

33. What is the area of the region bounded by the curves and x -axis?

- (a) 3 square units
(b) 4 square units
(c) 5 square units
(d) 6 square units

For the next two (2) items that follow :

Consider the function

$$f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$$

where p is a constant.

34. What is the value of $f'(0)$?

- (a) p^3 (b) $3p^3$
(c) $6p^3$ (d) $-6p^3$

35. What is the value of p for which $f''(0) = 0$?

- (a) $-\frac{1}{6}$ or 0
(b) -1 or 0
(c) $-\frac{1}{6}$ or 1
(d) -1 or 1

For the next two (2) items that follow :

Consider a triangle ABC in which

$$\cos A + \cos B + \cos C = \sqrt{3} \sin \frac{\pi}{3}$$

36. What is the value of $\sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$?

- (a) $\frac{1}{2}$ (b) $\frac{1}{4}$
(c) $\frac{1}{8}$ (d) $\frac{1}{16}$

37. What is the value of

$$\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{C+A}{2}\right)?$$

(a) $\frac{1}{4}$

(b) $\frac{1}{2}$

(c) $\frac{1}{16}$

(d) None of the above

For the next two (2) items that follow :

Given that $\tan\alpha$ and $\tan\beta$ are the roots of the equation $x^2 + bx + c = 0$ with $b \neq 0$.

38. What is $\tan(\alpha + \beta)$ equal to?

(a) $b(c-1)$

(b) $c(b-1)$

(c) $c(b-1)^{-1}$

(d) $b(c-1)^{-1}$

39. What is $\sin(\alpha + \beta) \sec\alpha \sec\beta$ equal to?

(a) b

(b) $-b$

(c) c

(d) $-c$

For the next two (2) items that follow :

Consider the two circles

$$(x-1)^2 + (y-3)^2 = r^2 \text{ and}$$

$$x^2 + y^2 - 8x + 2y + 8 = 0$$

40. What is the distance between the centres of the two circles?

(a) 5 units

(b) 6 units

(c) 8 units

(d) 10 units

41. If the circles intersect at two distinct points, then which one of the following is correct?

(a) $r = 1$

(b) $1 < r < 2$

(c) $r = 2$

(d) $2 < r < 8$

For the next two (2) items that follow :

Consider the two lines

$$x + y + 1 = 0 \text{ and } 3x + 2y + 1 = 0$$

42. What is the equation of the line passing through the point of intersection of the given lines and parallel to x -axis?

(a) $y + 1 = 0$

(b) $y - 1 = 0$

(c) $y - 2 = 0$

(d) $y + 2 = 0$

43. What is the equation of the line passing through the point of intersection of the given lines and parallel to y -axis?

(a) $x+1=0$

(b) $x-1=0$

(c) $x-2=0$

(d) $x+2=0$

For the next two (2) items that follow :

Consider the equation

$$k \sin x + \cos 2x = 2k - 7$$

44. If the equation possesses solution, then what is the minimum value of k ?

(a) 1

(b) 2

(c) 4

(d) 6

45. If the equation possesses solution, then what is the maximum value of k ?

(a) 1

(b) 2

(c) 4

(d) 6

For the next two (2) items that follow :

Consider the function

$$f(x) = \frac{a^{[x]+x} - 1}{[x] + x}$$

where $[\cdot]$ denotes the greatest integer function.

46. What is $\lim_{x \rightarrow 0^+} f(x)$ equal to?

(a) 1

(b) $\ln a$

(c) $1 - a^{-1}$

(d) Limit does not exist

47. What is $\lim_{x \rightarrow 0^-} f(x)$ equal to?

(a) 0

(b) $\ln a$

(c) $1 - a^{-1}$

(d) Limit does not exist

For the next two (2) items that follow :

Let z_1, z_2 and z_3 be non-zero complex numbers satisfying $z^2 = i\bar{z}$, where $i = \sqrt{-1}$.

48. What is $z_1 + z_2 + z_3$ equal to?

(a) i

(b) $-i$

(c) 0

(d) 1

49. Consider the following statements :

1. $z_1 z_2 z_3$ is purely imaginary.
2. $z_1 z_2 + z_2 z_3 + z_3 z_1$ is purely real.

Which of the above statements is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

For the next two (2) items that follow :

Given that $\log_x y$, $\log_z x$, $\log_y z$ are in GP, $xyz = 64$ and x^3, y^3, z^3 are in AP.

50. Which one of the following is correct?

x , y and z are

- (a) in AP only
- (b) in GP only
- (c) in both AP and GP
- (d) neither in AP nor in GP

51. Which one of the following is correct?

xy , yz and zx are

- (a) in AP only
- (b) in GP only
- (c) in both AP and GP
- (d) neither in AP nor in GP

For the next two (2) items that follow :

Let z be a complex number satisfying

$$\left| \frac{z-4}{z-8} \right| = 1 \text{ and } \left| \frac{z}{z-2} \right| = \frac{3}{2}$$

52. What is $|z|$ equal to?

- (a) 6
- (b) 12
- (c) 18
- (d) 36

53. What is $\left| \frac{z-6}{z+6} \right|$ equal to?

- (a) 3
- (b) 2
- (c) 1
- (d) 0

For the next two (2) items that follow :

A function $f(x)$ is defined as follows :

$$f(x) = \begin{cases} x + \pi & \text{for } x \in [-\pi, 0) \\ \pi \cos x & \text{for } x \in \left[0, \frac{\pi}{2}\right] \\ \left(x - \frac{\pi}{2}\right)^2 & \text{for } x \in \left(\frac{\pi}{2}, \pi\right] \end{cases}$$

54. Consider the following statements :

1. The function $f(x)$ is continuous at $x = 0$.
2. The function $f(x)$ is continuous at $x = \frac{\pi}{2}$.

Which of the above statements is/are correct?

- (a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

55. Consider the following statements :

1. The function $f(x)$ is differentiable at $x = 0$.
2. The function $f(x)$ is differentiable at $x = \frac{\pi}{2}$.

Which of the above statements is/are correct?

- (a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

For the next two (2) items that follow :

Let α and β ($\alpha < \beta$) be the roots of the equation $x^2 + bx + c = 0$, where $b > 0$ and $c < 0$.

56. Consider the following :

1. $\beta < -\alpha$
2. $\beta < |\alpha|$

Which of the above is/are correct?

- (a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

57. Consider the following :

1. $\alpha + \beta + \alpha\beta > 0$
2. $\alpha^2\beta + \beta^2\alpha > 0$

Which of the above is/are correct?

- (a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

For the next three (3) items that follow :

Consider a parallelogram whose vertices are $A(1, 2)$, $B(4, y)$, $C(x, 6)$ and $D(3, 5)$ taken in order.

58. What is the value of $AC^2 - BD^2$?

- (a) 25
- (b) 30
- (c) 36
- (d) 40

59. What is the point of intersection of the diagonals?

- (a) $\left(\frac{7}{2}, 4\right)$
- (b) $(3, 4)$
- (c) $\left(\frac{7}{2}, 5\right)$
- (d) $(3, 5)$

60. What is the area of the parallelogram?

- (a) $\frac{7}{2}$ square units
- (b) 4 square units
- (c) $\frac{11}{2}$ square units
- (d) 7 square units

For the next four (4) items that follow :

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that

$$f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$$

for $x \in \mathbb{R}$.

61. What is $f(1)$ equal to?

- (a) -2
- (b) -1
- (c) 0
- (d) 4

62. What is $f'(1)$ equal to?

- (a) -6
- (b) -5
- (c) 1
- (d) 0

63. What is $f'''(10)$ equal to?

- (a) 1
- (b) 5
- (c) 6
- (d) 8

64. Consider the following :

1. $f(2) = f(1) - f(0)$
2. $f''(2) - 2f'(1) = 12$

Which of the above is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

For the next three (3) items that follow :

A plane P passes through the line of intersection of the planes $2x - y + 3z = 2$, $x + y - z = 1$ and the point $(1, 0, 1)$.

65. What are the direction ratios of the line of intersection of the given planes?

- (a) $\langle 2, -5, -3 \rangle$
- (b) $\langle 1, -5, -3 \rangle$
- (c) $\langle 2, 5, 3 \rangle$
- (d) $\langle 1, 3, 5 \rangle$

66. What is the equation of the plane P ?

- (a) $2x + 5y - 2 = 0$
- (b) $5x + 2y - 5 = 0$
- (c) $x + z - 2 = 0$
- (d) $2x - y - 2z = 0$

67. If the plane P touches the sphere $x^2 + y^2 + z^2 = r^2$, then what is r equal to?

- (a) $\frac{2}{\sqrt{29}}$
- (b) $\frac{4}{\sqrt{29}}$
- (c) $\frac{5}{\sqrt{29}}$
- (d) 1

For the next two (2) items that follow :

Consider the function

$$f(x) = |x^2 - 5x + 6|$$

68. What is $f'(4)$ equal to?

- (a) -4
- (b) -3
- (c) 3
- (d) 2

69. What is $f''(2.5)$ equal to?

- (a) -3
- (b) -2
- (c) 0
- (d) 2

For the next two (2) items that follow :

Let $f(x)$ be the greatest integer function and $g(x)$ be the modulus function.

70. What is $(g \circ f)\left(-\frac{5}{3}\right) - (f \circ g)\left(-\frac{5}{3}\right)$ equal to?

- (a) -1
- (b) 0
- (c) 1
- (d) 2

71. What is $(f \circ f)\left(-\frac{9}{5}\right) + (g \circ g)(-2)$ equal to?

- (a) -1
- (b) 0
- (c) 1
- (d) 2

For the next two (2) items that follow :

Consider a circle passing through the origin and the points (a, b) and $(-b, -a)$.

72. On which line does the centre of the circle lie?

- (a) $x + y = 0$
- (b) $x - y = 0$
- (c) $x + y = a + b$
- (d) $x - y = a^2 - b^2$

73. What is the sum of the squares of the intercepts cut off by the circle on the axes?

(a) $\left(\frac{a^2 + b^2}{a^2 - b^2}\right)^2$

(b) $2\left(\frac{a^2 + b^2}{a - b}\right)^2$

(c) $4\left(\frac{a^2 + b^2}{a - b}\right)^2$

(d) None of the above

For the next two (2) items that follow :

Let $\hat{a}$, $\hat{b}$ be two unit vectors and θ be the angle between them.

74. What is $\cos\left(\frac{\theta}{2}\right)$ equal to?

(a) $\frac{|\hat{a} - \hat{b}|}{2}$

(b) $\frac{|\hat{a} + \hat{b}|}{2}$

(c) $\frac{|\hat{a} - \hat{b}|}{4}$

(d) $\frac{|\hat{a} + \hat{b}|}{4}$

75. What is $\sin\left(\frac{\theta}{2}\right)$ equal to?

(a) $\frac{|\hat{a} - \hat{b}|}{2}$

(b) $\frac{|\hat{a} + \hat{b}|}{2}$

(c) $\frac{|\hat{a} - \hat{b}|}{4}$

(d) $\frac{|\hat{a} + \hat{b}|}{4}$

76. Consider the following statements :

1. There exists $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ for which $\tan^{-1}(\tan \theta) \neq \theta$.

2. $\sin^{-1}\left(\frac{1}{3}\right) - \sin^{-1}\left(\frac{1}{5}\right) = \sin^{-1}\left(\frac{2\sqrt{2}(\sqrt{3}-1)}{15}\right)$

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

77. Consider the following statements :

1. $\tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right) = \pi$

2. There exist $x, y \in [-1, 1]$, where $x \neq y$ such that $\sin^{-1} x + \cos^{-1} y = \frac{\pi}{2}$.

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

78. What are the order and degree respectively of the differential equation whose solution is $y = cx + c^2 - 3c^{3/2} + 2$, where c is a parameter?

- (a) 1, 2
- (b) 2, 2
- (c) 1, 3
- (d) 1, 4

79. What is

$$\int_{-2}^2 x dx - \int_{-2}^2 [x] dx$$

equal to, where $[\cdot]$ is the greatest integer function?

- (a) 0
- (b) 1
- (c) 2
- (d) 4

80. If

$$\int_{-2}^5 f(x) dx = 4 \text{ and } \int_0^5 \{1 + f(x)\} dx = 7$$

then what is $\int_{-2}^0 f(x) dx$ equal to?

- (a) -3
- (b) 2
- (c) 3
- (d) 5

81. If $\lim_{x \rightarrow 0} \phi(x) = a^2$, where $a \neq 0$, then

what is $\lim_{x \rightarrow 0} \phi\left(\frac{x}{a}\right)$ equal to?

- (a) a^2
- (b) a^{-2}
- (c) $-a^2$
- (d) $-a$

82. What is $\lim_{x \rightarrow 0} e^{-\frac{1}{x^2}}$ equal to?

- (a) 0
- (b) 1
- (c) -1
- (d) Limit does not exist

83. If A is a square matrix, then what is $\text{adj}(A^{-1}) - (\text{adj } A)^{-1}$ equal to?

- (a) $2|A|$
- (b) Null matrix
- (c) Unit matrix
- (d) None of the above

84. What is the binary equivalent of the decimal number 0.3125?

- (a) 0.0111
- (b) 0.1010
- (c) 0.0101
- (d) 0.1101

85. Let R be a relation on the set N of natural numbers defined by ' $nRm \Leftrightarrow n$ is a factor of m '. Then which one of the following is correct?

- (a) R is reflexive, symmetric but not transitive
- (b) R is transitive, symmetric but not reflexive
- (c) R is reflexive, transitive but not symmetric
- (d) R is an equivalence relation

86. What is $\int_0^{4\pi} |\cos x| dx$ equal to?

(a) 0

(b) 2

(c) 4

(d) 8

87. What is the number of natural numbers less than or equal to 1000 which are neither divisible by 10 nor 15 nor 25?

(a) 860

(b) 854

(c) 840

(d) 824

88. $(a, 2b)$ is the mid-point of the line segment joining the points $(10, -6)$ and $(k, 4)$. If $a - 2b = 7$, then what is the value of k ?

(a) 2

(b) 3

(c) 4

(d) 5

89. Consider the following statements :

1. If ABC is an equilateral triangle, then $3 \tan(A+B) \tan C = 1$.

2. If ABC is a triangle in which $A = 78^\circ$, $B = 66^\circ$, then

$$\tan\left(\frac{A}{2} + C\right) < \tan A$$

3. If ABC is any triangle, then

$$\tan\left(\frac{A+B}{2}\right) \sin\left(\frac{C}{2}\right) < \cos\left(\frac{C}{2}\right)$$

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) 1 and 2

(d) 2 and 3

90. If $A = (\cos 12^\circ - \cos 36^\circ)(\sin 96^\circ + \sin 24^\circ)$ and $B = (\sin 60^\circ - \sin 12^\circ)(\cos 48^\circ - \cos 72^\circ)$, then what is $\frac{A}{B}$ equal to?

(a) -1

(b) 0

(c) 1

(d) 2

91. What is the mean deviation from the mean of the numbers 10, 9, 21, 16, 24 ?

(a) 5.2

(b) 5.0

(c) 4.5

(d) 4.0

92. Three dice are thrown simultaneously. What is the probability that the sum on the three faces is at least 5?

(a) $\frac{17}{18}$

(b) $\frac{53}{54}$

(c) $\frac{103}{108}$

(d) $\frac{215}{216}$

93. Two independent events A and B have $P(A) = \frac{1}{3}$ and $P(B) = \frac{3}{4}$. What is the probability that exactly one of the two events A or B occurs?

(a) $\frac{1}{4}$

(b) $\frac{5}{6}$

(c) $\frac{5}{12}$

(d) $\frac{7}{12}$

94. A coin is tossed three times. What is the probability of getting head and tail alternately?

(a) $\frac{1}{8}$

(b) $\frac{1}{4}$

(c) $\frac{1}{2}$

(d) $\frac{3}{4}$

95. If the total number of observations is 20, $\sum x_i = 1000$ and $\sum x_i^2 = 84000$, then what is the variance of the distribution?

(a) 1500

(b) 1600

(c) 1700

(d) 1800

96. A card is drawn from a well-shuffled deck of 52 cards. What is the probability that it is queen of spade?

(a) $\frac{1}{52}$

(b) $\frac{1}{13}$

(c) $\frac{1}{4}$

(d) $\frac{1}{8}$

97. If two dice are thrown, then what is the probability that the sum on the two faces is greater than or equal to 4?

(a) $\frac{13}{18}$

(b) $\frac{5}{6}$

(c) $\frac{11}{12}$

(d) $\frac{35}{36}$

98. A certain type of missile hits the target with probability $p = 0.3$. What is the least number of missiles should be fired so that there is at least an 80% probability that the target is hit?

(a) 5

(b) 6

(c) 7

(d) None of the above

99. For two mutually exclusive events A and B, $P(A) = 0.2$ and $P(\bar{A} \cap B) = 0.3$. What is $P(A | (A \cup B))$ equal to?

(a) $\frac{1}{2}$

(b) $\frac{2}{5}$

(c) $\frac{2}{7}$

(d) $\frac{2}{3}$

100. What is the probability of 5 Sundays in the month of December?

(a) $\frac{1}{7}$

(b) $\frac{2}{7}$

(c) $\frac{3}{7}$

(d) None of the above

101. If m is the geometric mean of

$$\left(\frac{y}{z}\right)^{\log(yz)}, \left(\frac{z}{x}\right)^{\log(zx)} \text{ and } \left(\frac{x}{y}\right)^{\log(xy)}$$

then what is the value of m ?

(a) 1

(b) 3

(c) 6

(d) 9

102. A point is chosen at random inside a rectangle measuring 6 inches by 5 inches. What is the probability that the randomly selected point is at least one inch from the edge of the rectangle?

(a) $\frac{2}{3}$

(b) $\frac{1}{3}$

(c) $\frac{1}{4}$

(d) $\frac{2}{5}$

103. The mean of the series $x_1, x_2, \dots, x_n$ is $\bar{X}$. If x_2 is replaced by λ , then what is the new mean?

- (a) $\bar{X} - x_2 + \lambda$
 (b) $\frac{\bar{X} - x_2 - \lambda}{n}$
 (c) $\frac{\bar{X} - x_2 + \lambda}{n}$
 (d) $\frac{n\bar{X} - x_2 + \lambda}{n}$

104. For the data

3, 5, 1, 6, 5, 9, 5, 2, 8, 6

the mean, median and mode are x , y and z respectively. Which one of the following is correct?

- (a) $x = y \neq z$
 (b) $x \neq y = z$
 (c) $x \neq y \neq z$
 (d) $x = y = z$

105. Consider the following statements in respect of a histogram :

- The total area of the rectangles in a histogram is equal to the total area bounded by the corresponding frequency polygon and the x -axis.
- When class intervals are unequal in a frequency distribution, the area of the rectangle is proportional to the frequency.

Which of the above statements is/are correct?

- (a) 1 only
 (b) 2 only
 (c) Both 1 and 2
 (d) Neither 1 nor 2

106. A fair coin is tossed 100 times. What is the probability of getting tails an odd number of times?

- (a) $\frac{1}{2}$
 (b) $\frac{3}{8}$
 (c) $\frac{1}{4}$
 (d) $\frac{1}{8}$

107. What is the number of ways in which 3 holiday travel tickets are to be given to 10 employees of an organization, if each employee is eligible for any one or more of the tickets?

- (a) 60
 (b) 120
 (c) 500
 (d) 1000

108. If one root of the equation

$$(l - m)x^2 + lx + 1 = 0$$

is double the other and l is real, then what is the greatest value of m ?

- (a) $-\frac{9}{8}$ (b) $\frac{9}{8}$
 (c) $-\frac{8}{9}$ (d) $\frac{8}{9}$

109. What is the number of four-digit decimal numbers (< 1) in which no digit is repeated?

- (a) 3024
 (b) 4536
 (c) 5040
 (d) None of the above

110. What is a vector of unit length orthogonal to both the vectors $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} + 3\hat{j} - \hat{k}$?

(a) $\frac{-4\hat{i} + 3\hat{j} - \hat{k}}{\sqrt{26}}$

(b) $\frac{-4\hat{i} + 3\hat{j} + \hat{k}}{\sqrt{26}}$

(c) $\frac{-3\hat{i} + 2\hat{j} - \hat{k}}{\sqrt{14}}$

(d) $\frac{-3\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{14}}$

111. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are the position vectors of the vertices of an equilateral triangle whose orthocentre is at the origin, then which one of the following is correct?

(a) $\vec{a} + \vec{b} + \vec{c} = \vec{0}$

(b) $\vec{a} + \vec{b} + \vec{c} = \text{unit vector}$

(c) $\vec{a} + \vec{b} = \vec{c}$

(d) $\vec{a} = \vec{b} + \vec{c}$

112. What is the area of the parallelogram having diagonals $3\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{i} - 3\hat{j} + 4\hat{k}$?

(a) $5\sqrt{5}$ square units

(b) $4\sqrt{5}$ square units

(c) $5\sqrt{3}$ square units

(d) $15\sqrt{2}$ square units

113. Consider the following in respect of the matrix $A = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$:

1. $A^2 = -A$

2. $A^3 = 4A$

Which of the above is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

114. Which of the following determinants have value 'zero'?

1. $\begin{vmatrix} 41 & 1 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix}$

2. $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$

3. $\begin{vmatrix} 0 & c & b \\ -c & 0 & a \\ -b & -a & 0 \end{vmatrix}$

Select the correct answer using the code given below.

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

115. What is the acute angle between the lines represented by the equations $y - \sqrt{3}x - 5 = 0$ and $\sqrt{3}y - x + 6 = 0$?

(a) 30°
(b) 45°
(c) 60°
(d) 75°

116. The system of linear equations $kx + y + z = 1$, $x + ky + z = 1$ and $x + y + kz = 1$ has a unique solution under which one of the following conditions?

(a) $k \neq 1$ and $k \neq -2$
(b) $k \neq 1$ and $k \neq 2$
(c) $k \neq -1$ and $k \neq -2$
(d) $k \neq -1$ and $k \neq 2$

117. What is the number of different messages that can be represented by three 0's and two 1's?

(a) 10
(b) 9
(c) 8
(d) 7

118. If $\log_a(ab) = x$, then what is $\log_b(ab)$ equal to?

(a) $\frac{1}{x}$
(b) $\frac{x}{x+1}$
(c) $\frac{x}{1-x}$
(d) $\frac{x}{x-1}$

119. If

$$y = \log_{10} x + \log_x 10 + \log_x x + \log_{10} 10$$

then what is

$$\left(\frac{dy}{dx}\right)_{x=10}$$

equal to?

(a) 10
(b) 2
(c) 1
(d) 0

120. Suppose ω_1 and ω_2 are two distinct cube roots of unity different from 1. Then what is $(\omega_1 - \omega_2)^2$ equal to?

(a) 3
(b) 1
(c) -1
(d) -3