2015

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO

T.B.C. : B–PFCA–O–PEW **Test Booklet Series**

Serial No.

0426177

TEST BOOKLET

A

MATHEMATICS

Time Allowed : Two Hours and Thirty Minutes **Maximum Marks : 300**

INSTRUCTIONS

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET *DOES NOT* HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS, ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.
2. **Please note that it is the candidate's responsibility to encode and fill in the Roll Number and Test Booklet Series A, B, C or D carefully and without any omission or discrepancy at the appropriate places in the OMR Answer Sheet. Any omission/ discrepancy will render the Answer Sheet liable for rejection.**
3. You have to enter your Roll Number on the Test Booklet in the Box provided alongside. *DO NOT* write *anything else* on the Test Booklet.
4. This Test Booklet contains **120** items (questions). Each item is printed both in **Hindi** and **English**. Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose *ONLY ONE* response for each item.
5. You have to mark all your responses *ONLY* on the separate Answer Sheet provided. See directions in the Answer Sheet.
6. ***All*** items carry equal marks.
7. Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions sent to you with your Admission Certificate.
8. After you have completed filling in all your responses on the Answer Sheet and the examination has concluded, you should hand over to the Invigilator *only the Answer Sheet*. You are permitted to take away with you the Test Booklet.
9. Sheets for rough work are appended in the Test Booklet at the end.
10. **Penalty for wrong answers :**
    THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
    (i) There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to that question will be deducted as penalty.
    (ii) If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.
    (iii) If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

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ध्यान दें : अनुदेशों का हिन्दी रूपान्तर इस पुस्तिका के पिछले पृष्ठ पर छपा है।

1. Let X be the set of all persons living in a city. Persons x, y in X are said to be related as $x < y$ if y is at least 5 years older than x. Which one of the following is correct?

   (a) The relation is an equivalence relation on X

   (b) The relation is transitive but neither reflexive nor symmetric

   (c) The relation is reflexive but neither transitive nor symmetric

   (d) The relation is symmetric but neither transitive nor reflexive

2. Which one of the following matrices is an elementary matrix?

   (a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 & 5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

   (c) $\begin{bmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

   (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 5 & 2 \end{bmatrix}$

3. Consider the following statements in respect of the given equation :

   $$(x^2+2)^2 + 8x^2 = 6x(x^2+2)$$

   1. All the roots of the equation are complex.
   2. The sum of all the roots of the equation is 6.

   Which of the above statements is/are correct?

   (a) 1 only

   (b) 2 only

   (c) Both 1 and 2

   (d) Neither 1 nor 2

4. In solving a problem that reduces to a quadratic equation, one student makes a mistake in the constant term and obtains 8 and 2 for roots. Another student makes a mistake only in the coefficient of first-degree term and finds -9 and -1 for roots. The correct equation is

   (a) $x^2 - 10x + 9 = 0$

   (b) $x^2 + 10x + 9 = 0$

   (c) $x^2 - 10x + 16 = 0$

   (d) $x^2 - 8x - 9 = 0$

5. If

$$A = \begin{bmatrix} 2 & 7 \\ 1 & 5 \end{bmatrix}$$

then what is $A + 3A^{-1}$ equal to?

(a) $3I$

(b) $5I$

(c) $7I$

(d) None of the above

where I is the identity matrix of order 2.

6. In a class of 60 students, 45 students like music, 50 students like dancing, 5 students like neither. Then the number of students in the class who like both music and dancing is

(a) 35

(b) 40

(c) 50

(d) 55

7. If

$\log_{10} 2$, $\log_{10}(2^x - 1)$ and $\log_{10}(2^x + 3)$

are three consecutive terms of an AP, then the value of x is

(a) 1

(b) $\log_5 2$

(c) $\log_2 5$

(d) $\log_{10} 5$

8. The matrix

$$\begin{bmatrix} 0 & -4+i \\ 4+i & 0 \end{bmatrix}$$

is

(a) symmetric

(b) skew-symmetric

(c) Hermitian

(d) skew-Hermitian

9. Let Z be the set of integers and aRb, where $a, b \in Z$ if and only if $(a - b)$ is divisible by 5.

Consider the following statements :

1. The relation R partitions Z into five equivalent classes.

2. Any two equivalent classes are either equal or disjoint.

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

10. If

$$z = \frac{-2(1+2i)}{3+i}$$

where $i = \sqrt{-1}$, then the argument $\theta(-\pi < \theta \leq \pi)$ of z is

(a) $\frac{3\pi}{4}$

(b) $\frac{\pi}{4}$

(c) $\frac{5\pi}{6}$

(d) $-\frac{3\pi}{4}$

11. If m and n are the roots of the equation $(x+p)(x+q)-k=0$, then the roots of the equation

$$(x-m)(x-n)+k=0$$

are

(a) p and q

(b) $1/p$ and $1/q$

(c) $-p$ and $-q$

(d) $p+q$ and $p-q$

12. What is the sum of the series $0\cdot5+0\cdot55+0\cdot555+\cdots$ to n terms?

(a) $\frac{5}{9}\left[n-\frac{2}{9}\left(1-\frac{1}{10^n}\right)\right]$

(b) $\frac{1}{9}\left[5-\frac{2}{9}\left(1-\frac{1}{10^n}\right)\right]$

(c) $\frac{1}{9}\left[n-\frac{5}{9}\left(1-\frac{1}{10^n}\right)\right]$

(d) $\frac{5}{9}\left[n-\frac{1}{9}\left(1-\frac{1}{10^n}\right)\right]$

13. If $1, \omega, \omega^2$ are the cube roots of unity, then the value of

$$(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$$

is

(a) -1

(b) 0

(c) 1

(d) 2

14. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Then the number of subsets of A containing exactly two elements is

(a) 20

(b) 40

(c) 45

(d) 90

15. What is the square root of i, where $i = \sqrt{-1}$?

(a) $\frac{1+i}{2}$

(b) $\frac{1-i}{2}$

(c) $\frac{1+i}{\sqrt{2}}$

(d) None of the above

16. The decimal number $(127 \cdot 25)_{10}$, when converted to binary number, takes the form

(a) $(1111111 \cdot 11)_2$

(b) $(1111110 \cdot 01)_2$

(c) $(1110111 \cdot 11)_2$

(d) $(1111111 \cdot 01)_2$

17. Consider the following in respect of two non-singular matrices A and B of same order :

1. $\det(A + B) = \det A + \det B$
2. $(A + B)^{-1} = A^{-1} + B^{-1}$

Which of the above is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

18. If

$$X = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}, \; B = \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} \text{ and } A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

satisfy the equation $AX = B$, then the matrix A is equal to

(a) $\begin{bmatrix} -7 & 26 \\ 1 & -5 \end{bmatrix}$ *(b)* $\begin{bmatrix} 7 & 26 \\ 4 & 17 \end{bmatrix}$

(c) $\begin{bmatrix} -7 & -4 \\ 26 & 13 \end{bmatrix}$ *(d)* $\begin{bmatrix} -7 & 26 \\ -6 & 23 \end{bmatrix}$

19. What is $\sum_{r=0}^{1} {}^{n+r}C_n$ equal to?

(a) ${}^{n+2}C_1$

(b) ${}^{n+2}C_n$

(c) ${}^{n+3}C_n$

(d) ${}^{n+2}C_{n+1}$

20. How many words can be formed using all the letters of the word 'NATION' so that all the three vowels should never come together?

(a) 354

(b) 348

(c) 288

(d) None of the above

21. $(x^3 - 1)$ can be factorised as

(a) $(x - 1)(x - \omega)(x + \omega^2)$

(b) $(x - 1)(x - \omega)(x - \omega^2)$

(c) $(x - 1)(x + \omega)(x + \omega^2)$

(d) $(x - 1)(x + \omega)(x - \omega^2)$

where ω is one of the cube roots of unity.

22. What is

$$\left[\frac{\sin\frac{\pi}{6}+i\left(1-\cos\frac{\pi}{6}\right)}{\sin\frac{\pi}{6}-i\left(1-\cos\frac{\pi}{6}\right)}\right]^3$$

where $i=\sqrt{-1}$, equal to?

(a) 1 *(b)* -1

(c) i *(d)* $-i$

23. Let

$$A=\begin{bmatrix}x+y & y\\ 2x & x-y\end{bmatrix},\ B=\begin{bmatrix}2\\-1\end{bmatrix} \text{ and } C=\begin{bmatrix}3\\2\end{bmatrix}$$

If $AB = C$, then what is A^2 equal to?

(a) $\begin{bmatrix}6 & -10\\ 4 & 26\end{bmatrix}$

(b) $\begin{bmatrix}-10 & 5\\ 4 & 24\end{bmatrix}$

(c) $\begin{bmatrix}-5 & -6\\ -4 & -20\end{bmatrix}$

(d) $\begin{bmatrix}-5 & -7\\ -5 & 20\end{bmatrix}$

24. The value of

$$\begin{vmatrix}1 & 1 & 1\\ 1 & 1+x & 1\\ 1 & 1 & 1+y\end{vmatrix}$$

is

(a) $x+y$

(b) $x-y$

(c) xy

(d) $1+x+y$

25. If $A=\{x: x \text{ is a multiple of } 3\}$ and $B=\{x: x \text{ is a multiple of } 4\}$ and $C=\{x: x \text{ is a multiple of } 12\}$, then which one of the following is a null set?

(a) $(A \setminus B)\cup C$

(b) $(A \setminus B)\setminus C$

(c) $(A\cap B)\cap C$

(d) $(A\cap B)\setminus C$

26. If $(11101011)_2$ is converted to decimal system, then the resulting number is

(a) 235

(b) 175

(c) 160

(d) 126

27. What is the real part of

$$(\sin x+i\cos x)^3$$

where $i=\sqrt{-1}$?

(a) $-\cos 3x$

(b) $-\sin 3x$

(c) $\sin 3x$

(d) $\cos 3x$

28. If

$$E(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

then $E(\alpha)\,E(\beta)$ is equal to

(a) $E(\alpha\beta)$

(b) $E(\alpha - \beta)$

(c) $E(\alpha + \beta)$

(d) $-E(\alpha + \beta)$

29. Let $A = \{x, y, z\}$ and $B = \{p, q, r, s\}$. What is the number of distinct relations from B to A?

(a) 4096

(b) 4094

(c) 128

(d) 126

30. If

$2p + 3q = 18$

and $4p^2 + 4pq - 3q^2 - 36 = 0$

then what is $(2p + q)$ equal to?

(a) 6

(b) 7

(c) 10

(d) 20

31. Let θ be a positive angle. If the number of degrees in θ is divided by the number of radians in θ, then an irrational number $180/\pi$ results. If the number of degrees in θ is multiplied by the number of radians in θ, then an irrational number $125\pi/9$ results. The angle θ must be equal to

(a) 30°

(b) 45°

(c) 50°

(d) 60°

32. In a triangle ABC, $a = (1 + \sqrt{3})$ cm, $b = 2$ cm and angle $C = 60°$. Then the other two angles are

(a) 45° and 75°

(b) 30° and 90°

(c) 105° and 15°

(d) 100° and 20°

For the next two (2) items that follow :

Let α be the root of the equation $25\cos^2\theta + 5\cos\theta - 12 = 0$, where $\frac{\pi}{2} < \alpha < \pi$.

33. What is $\tan\alpha$ equal to?

(a) $-3/4$

(b) $3/4$

(c) $-4/3$

(d) $-4/5$

34. What is $\sin 2\alpha$ equal to?

(a) 24/25

(b) −24/25

(c) −5/12

(d) −21/25

35. The angle of elevation of the top of a tower from a point 20 m away from its base is 45°. What is the height of the tower?

(a) 10 m

(b) 20 m

(c) 30 m

(d) 40 m

36. The equation

$$\tan^{-1}(1+x)+\tan^{-1}(1-x)=\frac{\pi}{2}$$

is satisfied by

(a) $x = 1$

(b) $x = -1$

(c) $x = 0$

(d) $x = 1/2$

37. The angles of elevation of the top of a tower standing on a horizontal plane from two points on a line passing through the foot of the tower at distances 49 m and 36 m are 43° and 47° respectively. What is the height of the tower?

(a) 40 m

(b) 42 m

(c) 45 m

(d) 47 m

38. $(1-\sin A+\cos A)^2$ is equal to

(a) $2(1-\cos A)(1+\sin A)$

(b) $2(1-\sin A)(1+\cos A)$

(c) $2(1-\cos A)(1-\sin A)$

(d) None of the above

39. What is

$$\frac{\cos\theta}{1-\tan\theta}+\frac{\sin\theta}{1-\cot\theta}$$

equal to?

(a) $\sin\theta-\cos\theta$

(b) $\sin\theta+\cos\theta$

(c) $2\sin\theta$

(d) $2\cos\theta$

For the next three (3) items that follow :

Consider $x = 4\tan^{-1}\left(\frac{1}{5}\right)$, $y = \tan^{-1}\left(\frac{1}{70}\right)$ and $z = \tan^{-1}\left(\frac{1}{99}\right)$.

40. What is x equal to?

(a) $\tan^{-1}\left(\frac{60}{119}\right)$

(b) $\tan^{-1}\left(\frac{120}{119}\right)$

(c) $\tan^{-1}\left(\frac{90}{169}\right)$

(d) $\tan^{-1}\left(\frac{170}{169}\right)$

41. What is $x - y$ equal to?

(a) $\tan^{-1}\left(\frac{828}{845}\right)$

(b) $\tan^{-1}\left(\frac{8287}{8450}\right)$

(c) $\tan^{-1}\left(\frac{8281}{8450}\right)$

(d) $\tan^{-1}\left(\frac{8287}{8471}\right)$

42. What is $x - y + z$ equal to?

(a) $\frac{\pi}{2}$

(b) $\frac{\pi}{3}$

(c) $\frac{\pi}{6}$

(d) $\frac{\pi}{4}$

For the next three (3) items that follow :

Consider the triangle ABC with vertices $A(-2, 3)$, $B(2, 1)$ and $C(1, 2)$.

43. What is the circumcentre of the triangle ABC?

(a) $(-2, -2)$

(b) $(2, 2)$

(c) $(-2, 2)$

(d) $(2, -2)$

44. What is the centroid of the triangle ABC?

(a) $\left(\frac{1}{3}, 1\right)$

(b) $\left(\frac{1}{3}, 2\right)$

(c) $\left(1, \frac{2}{3}\right)$

(d) $\left(\frac{1}{2}, 3\right)$

45. What is the foot of the altitude from the vertex A of the triangle ABC?

(a) $(1, 4)$

(b) $(-1, 3)$

(c) $(-2, 4)$

(d) $(-1, 4)$

46. The point on the parabola $y^2 = 4ax$ nearest to the focus has its abscissa

(a) $x = 0$

(b) $x = a$

(c) $x = a/2$

(d) $x = 2a$

47. A line passes through (2, 2) and is perpendicular to the line $3x + y = 3$. Its y-intercept is

(a) 3/4

(b) 4/3

(c) 1/3

(d) 3

48. The hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ passes through the point $(3\sqrt{5}, 1)$ and the length of its latus rectum is $\frac{4}{3}$ units. The length of the conjugate axis is

(a) 2 units

(b) 3 units

(c) 4 units

(d) 5 units

49. The perpendicular distance between the straight lines $6x + 8y + 15 = 0$ and $3x + 4y + 9 = 0$ is

(a) 3/2 units

(b) 3/10 unit

(c) 3/4 unit

(d) 2/7 unit

50. The area of a triangle, whose vertices are (3, 4), (5, 2) and the point of intersection of the lines $x = a$ and $y = 5$, is 3 square units. What is the value of a?

(a) 2

(b) 3

(c) 4

(d) 5

51. The length of perpendicular from the origin to a line is 5 units and the line makes an angle 120° with the positive direction of x-axis. The equation of the line is

(a) $x + \sqrt{3}y = 5$

(b) $\sqrt{3}x + y = 10$

(c) $\sqrt{3}x - y = 10$

(d) None of the above

52. The equation of the line joining the origin to the point of intersection of the lines $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{b}+\frac{y}{a}=1$ is

(a) $x-y=0$

(b) $x+y=0$

(c) $x=0$

(d) $y=0$

For the next two (2) items that follow :

The projections of a directed line segment on the coordinate axes are 12, 4, 3 respectively.

53. What is the length of the line segment?

(a) 19 units

(b) 17 units

(c) 15 units

(d) 13 units

54. What are the direction cosines of the line segment?

(a) $\langle 12/13,\ 4/13,\ 3/13 \rangle$

(b) $\langle 12/13,\ -4/13,\ 3/13 \rangle$

(c) $\langle 12/13,\ -4/13,\ -3/13 \rangle$

(d) $\langle -12/13,\ -4/13,\ 3/13 \rangle$

For the next two (2) items that follow :

From the point $P(3,\ -1,\ 11)$, a perpendicular is drawn on the line L given by the equation $\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}$. Let Q be the foot of the perpendicular.

55. What are the direction ratios of the line segment PQ?

(a) $\langle 1,\ 6,\ 4 \rangle$

(b) $\langle -1,\ 6,\ -4 \rangle$

(c) $\langle -1,\ -6,\ 4 \rangle$

(d) $\langle 2,\ -6,\ 4 \rangle$

56. What is the length of the line segment PQ?

(a) $\sqrt{47}$ units

(b) 7 units

(c) $\sqrt{53}$ units

(d) 8 units

For the next two (2) items that follow :

A triangular plane ABC with centroid $(1,\ 2,\ 3)$ cuts the coordinate axes at A, B, C respectively.

57. What are the intercepts made by the plane ABC on the axes?

(a) 3, 6, 9

(b) 1, 2, 3

(c) 1, 4, 9

(d) 2, 4, 6

58. What is the equation of the plane ABC?

(a) $x + 2y + 3z = 1$

(b) $3x + 2y + z = 3$

(c) $2x + 3y + 6z = 18$

(d) $6x + 3y + 2z = 18$

For the next two (2) items that follow :

A point $P(1, 2, 3)$ is one vertex of a cuboid formed by the coordinate planes and the planes passing through P and parallel to the coordinate planes.

59. What is the length of one of the diagonals of the cuboid?

(a) $\sqrt{10}$ units

(b) $\sqrt{14}$ units

(c) 4 units

(d) 5 units

60. What is the equation of the plane passing through $P(1, 2, 3)$ and parallel to xy-plane?

(a) $x + y = 3$

(b) $x - y = -1$

(c) $z = 3$

(d) $x + 2y + 3z = 14$

61. If $G(x) = \sqrt{25 - x^2}$, then what is $\lim_{x \to 1} \frac{G(x) - G(1)}{x - 1}$ equal to?

(a) $-\frac{1}{2\sqrt{6}}$

(b) $\frac{1}{5}$

(c) $-\frac{1}{\sqrt{6}}$

(d) $\frac{1}{\sqrt{6}}$

62. Consider the following statements :

1. $y = \frac{e^x + e^{-x}}{2}$ is an increasing function on $[0, \infty)$.

2. $y = \frac{e^x - e^{-x}}{2}$ is an increasing function on $(-\infty, \infty)$.

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

63. For each non-zero real number x, let $f(x) = \frac{x}{|x|}$. The range of f is

(a) a null set

(b) a set consisting of only one element

(c) a set consisting of two elements

(d) a set consisting of infinitely many elements

64. Consider the following statements :

1. $f(x) = [x]$, where $[.]$ is the greatest integer function, is discontinuous at $x = n$, where $n \in Z$.
2. $f(x) = \cot x$ is discontinuous at $x = n\pi$, where $n \in Z$.

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

65. What is the derivative of

$$\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$$

with respect to $\tan^{-1} x$?

(a) 0

(b) 1/2

(c) 1

(d) x

66. If $f(x) = \log_e\left(\frac{1+x}{1-x}\right)$, $g(x) = \frac{3x + x^3}{1+3x^2}$ and $g \circ f(t) = g(f(t))$, then what is $g \circ f\left(\frac{e-1}{e+1}\right)$ equal to?

(a) 2

(b) 1

(c) 0

(d) 1/2

For the next two (2) items that follow :

Given a function

$$f(x) = \begin{cases} -1 & \text{if} \quad x \le 0 \\ ax + b & \text{if} \quad 0 < x < 1 \\ 1 & \text{if} \quad x \ge 1 \end{cases}$$

where a, b are constants. The function is continuous everywhere.

67. What is the value of a?

(a) -1

(b) 0

(c) 1

(d) 2

68. What is the value of b?

(a) -1

(b) 1

(c) 0

(d) 2

69. Consider the following functions :

1. $f(x) = x^3$, $x \in \mathbb{R}$
2. $f(x) = \sin x$, $0 < x < 2\pi$
3. $f(x) = e^x$, $x \in \mathbb{R}$

Which of the above functions have inverse defined on their ranges?

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

For the next two (2) items that follow :

The integral $\int \frac{dx}{a\cos x + b\sin x}$ is of the form $\frac{1}{r}\ln\left[\tan\left(\frac{x+\alpha}{2}\right)\right]$.

70. What is r equal to?

(a) $a^2 + b^2$

(b) $\sqrt{a^2 + b^2}$

(c) $a + b$

(d) $\sqrt{a^2 - b^2}$

71. What is α equal to?

(a) $\tan^{-1}\left(\frac{a}{b}\right)$

(b) $\tan^{-1}\left(\frac{b}{a}\right)$

(c) $\tan^{-1}\left(\frac{a+b}{a-b}\right)$

(d) $\tan^{-1}\left(\frac{a-b}{a+b}\right)$

For the next two (2) items that follow :

Consider the function $f(x) = \frac{x^2 - 1}{x^2 + 1}$, where $x \in \mathbb{R}$.

72. At what value of x does $f(x)$ attain minimum value?

(a) -1

(b) 0

(c) 1

(d) 2

73. What is the minimum value of $f(x)$?

(a) 0

(b) 1/2

(c) -1

(d) 2

For the next two (2) items that follow :

Consider the function

$$f(x) = \begin{cases} \frac{\alpha\cos x}{\pi - 2x} & \text{if} \quad x \neq \frac{\pi}{2} \\ 3 & \text{if} \quad x = \frac{\pi}{2} \end{cases}$$

which is continuous at $x = \frac{\pi}{2}$, where α is a constant.

74. What is the value of α?

(a) 6

(b) 3

(c) 2

(d) 1

75. What is $\lim_{x \to 0} f(x)$ equal to?

(a) 0

(b) 3

(c) $\frac{3}{\pi}$

(d) $\frac{6}{\pi}$

For the next two (2) items that follow :

Consider the line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$.

76. What is the area of the region in the first quadrant enclosed by the x-axis, the line $x = \sqrt{3}$ and the circle?

(a) $\frac{\pi}{3} - \frac{\sqrt{3}}{2}$

(b) $\frac{\pi}{2} - \frac{\sqrt{3}}{2}$

(c) $\frac{\pi}{3} - \frac{1}{2}$

(d) None of the above

77. What is the area of the region in the first quadrant enclosed by the x-axis, the line $x = \sqrt{3}y$ and the circle?

(a) $\frac{\pi}{3}$

(b) $\frac{\pi}{6}$

(c) $\frac{\pi}{3} - \frac{\sqrt{3}}{2}$

(d) None of the above

For the next two (2) items that follow :

Consider the curves $y = \sin x$ and $y = \cos x$.

78. What is the area of the region bounded by the above two curves and the lines $x = 0$ and $x = \frac{\pi}{4}$?

(a) $\sqrt{2} - 1$

(b) $\sqrt{2} + 1$

(c) $\sqrt{2}$

(d) 2

79. What is the area of the region bounded by the above two curves and the lines $x = \frac{\pi}{4}$ and $x = \frac{\pi}{2}$?

(a) $\sqrt{2} - 1$

(b) $\sqrt{2} + 1$

(c) $2\sqrt{2}$

(d) 2

For the next two (2) items that follow :

Consider the function

$$f(x) = 0 \cdot 75x^4 - x^3 - 9x^2 + 7$$

80. What is the maximum value of the function?

(a) 1

(b) 3

(c) 7

(d) 9

81. Consider the following statements :

1. The function attains local minima at $x = -2$ and $x = 3$.
2. The function increases in the interval $(-2, 0)$.

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

For the next three (3) items that follow :

Consider the parametric equation

$$x = \frac{a(1-t^2)}{1+t^2}, \quad y = \frac{2at}{1+t^2}$$

82. What does the equation represent?

(a) It represents a circle of diameter a

(b) It represents a circle of radius a

(c) It represents a parabola

(d) None of the above

83. What is $\frac{dy}{dx}$ equal to?

(a) $\frac{y}{x}$

(b) $-\frac{y}{x}$

(c) $\frac{x}{y}$

(d) $-\frac{x}{y}$

84. What is $\frac{d^2y}{dx^2}$ equal to?

(a) $\frac{a^2}{y^2}$ *(b)* $\frac{a^2}{x^2}$

(c) $-\frac{a^2}{x^2}$ *(d)* $-\frac{a^2}{y^3}$

85. Consider the following statements :

1. The general solution of
$$\frac{dy}{dx} = f(x) + x$$
is of the form $y = g(x) + c$, where c is an arbitrary constant.
2. The degree of $\left(\frac{dy}{dx}\right)^2 = f(x)$ is 2.

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

86. What is $\int \frac{dx}{\sqrt{x^2+a^2}}$ equal to?

(a) $\ln\left|\frac{x+\sqrt{x^2+a^2}}{a}\right|+c$

(b) $\ln\left|\frac{x-\sqrt{x^2+a^2}}{a}\right|+c$

(c) $\ln\left|\frac{x^2+\sqrt{x^2+a^2}}{a}\right|+c$

(d) None of the above

where c is the constant of integration.

For the next four (4) items that follow :

Consider the integral $I_m = \int_0^{\pi} \frac{\sin 2mx}{\sin x} dx$, where m is a positive integer.

87. What is I_1 equal to?

(a) 0

(b) 1/2

(c) 1

(d) 2

88. What is $I_2 + I_3$ equal to?

(a) 4

(b) 2

(c) 1

(d) 0

89. What is I_m equal to?

(a) 0

(b) 1

(c) m

(d) $2m$

90. Consider the following :

1. $I_m - I_{m-1}$ is equal to 0.
2. $I_{2m} > I_m$

Which of the above is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

For the next two (2) items that follow :

Given that $\frac{d}{dx}\left(\frac{1+x^2+x^4}{1+x+x^2}\right) = Ax + B.$

91. What is the value of A?

(a) -1

(b) 1

(c) 2

(d) 4

92. What is the value of B?

(a) -1

(b) 1

(c) 2

(d) 4

For the next two (2) items that follow :

Given that $\lim_{x\to\infty}\left(\frac{2+x^2}{1+x} - Ax - B\right) = 3.$

93. What is the value of A?

(a) -1

(b) 1

(c) 2

(d) 3

94. What is the value of B?

(a) -1

(b) 3

(c) -4

(d) -3

95. What is the solution of the differential equation

$$\frac{ydx - xdy}{y^2} = 0?$$

(a) $xy = c$

(b) $y = cx$

(c) $x + y = c$

(d) $x - y = c$

where c is an arbitrary constant.

96. What is the solution of the differential equation

$$\sin\left(\frac{dy}{dx}\right) - a = 0?$$

(a) $y = x\sin^{-1} a + c$

(b) $x = y\sin^{-1} a + c$

(c) $y = x + x\sin^{-1} a + c$

(d) $y = \sin^{-1} a + c$

where c is an arbitrary constant.

97. What is the solution of the differential equation

$$\frac{dx}{dy} + \frac{x}{y} - y^2 = 0?$$

(a) $xy = x^4 + c$

(b) $xy = y^4 + c$

(c) $4xy = y^4 + c$

(d) $3xy = y^3 + c$

where c is an arbitrary constant.

98. What is $\int \frac{xe^x dx}{(x+1)^2}$ equal to?

(a) $(x+1)^2 e^x + c$

(b) $(x+1)e^x + c$

(c) $\frac{e^x}{x+1} + c$

(d) $\frac{e^x}{(x+1)^2} + c$

where c is the constant of integration.

99. The adjacent sides AB and AC of a triangle ABC are represented by the vectors $-2\hat{i} + 3\hat{j} + 2\hat{k}$ and $-4\hat{i} + 5\hat{j} + 2\hat{k}$ respectively. The area of the triangle ABC is

(a) 6 square units

(b) 5 square units

(c) 4 square units

(d) 3 square units

100. A force $\vec{F} = 3\hat{i} + 4\hat{j} - 3\hat{k}$ is applied at the point P, whose position vector is $\vec{r} = 2\hat{i} - 2\hat{j} - 3\hat{k}$. What is the magnitude of the moment of the force about the origin?

(a) 23 units

(b) 19 units

(c) 18 units

(d) 21 units

101. Given that the vectors $\vec{\alpha}$ and $\vec{\beta}$ are non-collinear. The values of x and y for which $\vec{u} - \vec{v} = \vec{w}$ holds true if $\vec{u} = 2x\vec{\alpha} + y\vec{\beta}$, $\vec{v} = 2y\vec{\alpha} + 3x\vec{\beta}$ and $\vec{w} = 2\vec{\alpha} - 5\vec{\beta}$, are

(a) $x = 2,\ y = 1$

(b) $x = 1,\ y = 2$

(c) $x = -2,\ y = 1$

(d) $x = -2,\ y = -1$

102. If $|\vec{a}| = 7$, $|\vec{b}| = 11$ and $|\vec{a} + \vec{b}| = 10\sqrt{3}$, then $|\vec{a} - \vec{b}|$ is equal to

(a) 40

(b) 10

(c) $4\sqrt{10}$

(d) $2\sqrt{10}$

103. Let α, β, γ be distinct real numbers. The points with position vectors $\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$, $\beta\hat{i}+\gamma\hat{j}+\alpha\hat{k}$ and $\gamma\hat{i}+\alpha\hat{j}+\beta\hat{k}$

(a) are collinear

(b) form an equilateral triangle

(c) form a scalene triangle

(d) form a right-angled triangle

104. If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then which of the following is/are correct?

1. $\vec{a}, \vec{b}, \vec{c}$ are coplanar.
2. $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

Select the correct answer using the code given below.

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

105. If $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$, then which one of the following is correct?

(a) $\vec{a}=\lambda\vec{b}$ for some scalar λ

(b) $\vec{a}$ is parallel to $\vec{b}$

(c) $\vec{a}$ is perpendicular to $\vec{b}$

(d) $\vec{a}=\vec{b}=\vec{0}$

106. The mean and the variance of 10 observations are given to be 4 and 2 respectively. If every observation is multiplied by 2, the mean and the variance of the new series will be respectively

(a) 8 and 20

(b) 8 and 4

(c) 8 and 8

(d) 80 and 40

107. Which one of the following measures of central tendency is used in construction of index numbers?

(a) Harmonic mean

(b) Geometric mean

(c) Median

(d) Mode

108. The correlation coefficient between two variables X and Y is found to be 0·6. All the observations on X and Y are transformed using the transformations $U=2-3X$ and $V=4Y+1$. The correlation coefficient between the transformed variables U and V will be

(a) $-0\cdot5$

(b) $+0\cdot5$

(c) $-0\cdot6$

(d) $+0\cdot6$

109. Which of the following statements is/are correct in respect of regression coefficients?

1. It measures the degree of linear relationship between two variables.
2. It gives the value by which one variable changes for a unit change in the other variable.

Select the correct answer using the code given below.

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

110. A set of annual numerical data, comparable over the years, is given for the last 12 years.

Consider the following statements :

1. The data is best represented by a broken line graph, each corner (turning point) representing the data of one year.
2. Such a graph depicts the chronological change and also enables one to make a short-term forecast.

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

111. Two men hit at a target with probabilities 1/2 and 1/3 respectively. What is the probability that exactly one of them hits the target?

(a) 1/2

(b) 1/3

(c) 1/6

(d) 2/3

112. Two similar boxes B_i $(i = 1, 2)$ contain $(i + 1)$ red and $(5 - i - 1)$ black balls. One box is chosen at random and two balls are drawn randomly. What is the probability that both the balls are of different colours?

(a) 1/2

(b) 3/10

(c) 2/5

(d) 3/5

113. In an examination, the probability of a candidate solving a question is 1/2. Out of given 5 questions in the examination, what is the probability that the candidate was able to solve at least 2 questions?

(a) 1/64

(b) 3/16

(c) 1/2

(d) 13/16

114. If $A \subseteq B$, then which one of the following is ***not*** correct?

(a) $P(A \cap \overline{B}) = 0$

(b) $P(A \mid B) = \dfrac{P(A)}{P(B)}$

(c) $P(B \mid A) = \dfrac{P(B)}{P(A)}$

(d) $P(A \mid (A \cup B)) = \dfrac{P(A)}{P(B)}$

115. The mean and the variance in a binomial distribution are found to be 2 and 1 respectively. The probability $P(X = 0)$ is

(a) 1/2

(b) 1/4

(c) 1/8

(d) 1/16

116. The mean of five numbers is 30. If one number is excluded, their mean becomes 28. The excluded number is

(a) 28

(b) 30

(c) 35

(d) 38

117. If A and B are two events such that $P(A \cup B) = \dfrac{3}{4}$, $P(A \cap B) = \dfrac{1}{4}$ and $P(\overline{A}) = \dfrac{2}{3}$, then what is $P(B)$ equal to?

(a) 1/3

(b) 2/3

(c) 1/8

(d) 2/9

118. The 'less than' ogive curve and the 'more than' ogive curve intersect at

(a) median

(b) mode

(c) arithmetic mean

(d) None of the above

119. In throwing of two dice, the number of exhaustive events that '5' will never appear on any one of the dice is

(a) 5

(b) 18

(c) 25

(d) 36

120. Two cards are drawn successively without replacement from a well-shuffled pack of 52 cards. The probability of drawing two aces is

(a) 1/26

(b) 1/221

(c) 4/223

(d) 1/13